One DOF Climber Notes

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The one degree-of-freedom climber is an example cooked up between myself, Mark, and Dan Santos about what a really simple climber could look like:

The climber consists of a body connected to a (mass-less) foot via a spring, damper, and force actuator in parallel. The foot can clamp onto a vertical pole with a given clamping (normal) force, or release the pole. Gravity acts down the pole:

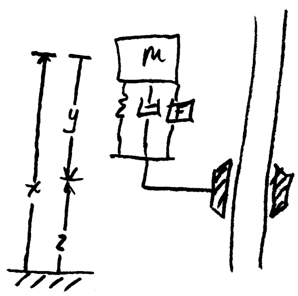

Figure 1: One-Dimensional Climber

There is Coulumb friction between the clamp and the pole when the climber has clamped the pole. The static friction limit is slightly more than the weight of the climber ($F_s \leq mg + \delta$, $\delta > 0$), and the kinetic friction force generated when sliding is slightly less than the weight of the climber ($F_d = mg + \zeta$, $\zeta < 0$).

Variables:
x - absolute coordinate of the body
y - relative coordinate from body to foot
y_n - uncompressed length of spring
z - absolute coordinate of the foot, $z = x - y$
r - amount of spring extension, $r = y - y_n$
m - mass of climber
g - gravity

F - actuator force
F_w - wall force applied to foot (friction force)
k - spring constant
b - damping constant

Equations when clamped and not sliding:
$\dot{r} = \dot{x}$
$\ddot{x} = -\frac{k}{m}r - \frac{b}{m}\dot{x} + \frac{F}{m} - g$
$y = r + y_n$
$\dot{y} = \dot{x}$
$z = x - r - y_n$
$\dot{z} = 0$
$F_w = -kr - b\dot{x} + F$

Equations when not clamped:
$\dot{r} = -\frac{k}{b}r + \frac{F}{b}$
$\ddot{x} = -g$
$y = r + y_n$
$\dot{y} = -\frac{k}{b}r + \frac{F}{b}$
$z = x - r - y_n$
$\dot{z} = \frac{k}{b}r + \dot{x} - \frac{F}{b}$
$F_w = 0$

Equations when sliding down:
$\dot{r} = -\frac{k}{b}r + \frac{F-2mg-\zeta}{b}$
$\ddot{x} = \frac{\zeta}{m}$
$y = r + y_n$
$\dot{y} = -\frac{k}{b}r + \frac{F-2mg-\zeta}{b}$
$z = x - r - y_n$
$\dot{z} = \frac{k}{b}r + \dot{x} + \frac{2mg+\zeta-F}{b}$
$F_w = mg + \zeta$

Equations when sliding up:
$\dot{r} = -\frac{k}{b}r + \frac{F-\zeta}{b}$
$\ddot{x} = -2g - \frac{\zeta}{m}$
$y = r + y_n$
$\dot{y} = -\frac{k}{b}r + \frac{F-\zeta}{b}$
$z = x - r - y_n$
$\dot{z} = \frac{k}{b}r + \dot{x} + \frac{\zeta-F}{b}$
$F_w = -mg - \zeta$

Transitions:
From clamped to free when $\dot{x} \leq 0$ and $r > 0$, set $F := -|F|$ (retract foot)

From clamped to sliding down when $F_w > mg + \delta$, set $\dot{x} := \dot{x} - \varepsilon$, $\dot{r} := \dot{r} + \varepsilon$ (give foot a small downward velocity), and $F := \mathrm{sign}(\dot{x})\,|F|$ (try to make $\dot{z}$ positive)
From clamped to sliding up when $F_w < -mg - \delta$, set $\dot{x} := \dot{x} + \varepsilon$, $\dot{r} := \dot{r} - \varepsilon$ (give foot a small downward velocity), and $F := -|F|$ (retract foot)
From free to clamped when $\dot{z} \leq 0$, set $\dot{z} := 0$, $\dot{r} := \dot{x}$, and $F := |F|$ (extend foot)
From sliding down to clamped when $|\dot{z}| < \varepsilon$, set $\dot{r} := \dot{x}$ and $F := |F|$ (extend foot)
From sliding up to clamped when $|\dot{z}| < \varepsilon$, set $\dot{r} := \dot{x}$ and $F := |F|$ (extend foot)

Some observations:
$F < mg + \delta$ to prevent sliding when you apply the force when the entire system is at rest.
The force applications given above for each state are intended in all cases to get (or keep) the foot moving up the pole.

To do:

- Put up some drawings in here
- Simulate a system with no friction limit and look at trajectories (use Matlab's LMI??)
- Simulate system with friction limits and look at trajectories