Design and Development of the Long-Jumping ”Grillo” Mini Robot

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Abstract—This paper describes the design of a fast long-jumping robot conceived to move in unstructured environments through simple feed-forward control laws. Despite the apparent similarities with hopping, jumping dynamics is peculiar and involve non-trivial issues on actuation powering, energy saving and stability. The ”Grillo” robot described here is a quadruped, 50-mm robot that weights about 15 grams and is suited for a long-jumping gait. Inspired by frog locomotion, a tiny motor load the springs connected to the hind limbs. At take-off, an escapement mechanism releases the loaded springs. This provides a peak power output that can exceed several times the maximum motor power. In this way, the actuation and energy systems can be significantly reduced in weight and size. On the other hand, passive dynamics is exploited by compliant forelegs, that let to partially recover the impact energy in their elastic recoil. Equipped with a 0.2W DC motor, the robot is dimensioned to achieve a forward speed of 1.5 m/s, which corresponds to about 30 body length per second.

I. INTRODUCTION

The robot presented in this paper suggests a possible solution for an efficient fast locomotion in unstructured terrains, where mini robots are often adopted in environment monitoring and exploration. In addition, based on nature observation, the robot itself is conceived as a physical model to verify the influence of scale effects in animal locomotion.

Small running robots, mainly hexapods, can achieve high performances by high frequency movements that mimic fast running insects [9],[10]. Indeed, our design is based on the idea that the optimal locomotion strategy strongly depends upon robot dimensions.

The size-dependent relation between locomotion strategy and gait efficiency can be easily noticed in nature, where different sized animals adopt very different gaits. In particular smaller running animals adopt longer step lengths compared to their leg length, with lower duty factors (the rate of ground contact time respect to overall step time) respect to bigger animals [3],[4].

On the other hand, gait similarities along very different dimensions provide interesting insights on gait dynamics. It has been observed that different sized animals switch from walk to run and from trot to gallop at about the same Froude number $F_r \approx 0.5$ [1]. Defined as $F_r = \frac{v^2}{gl}$, the Froude number can be considered as the ratio between kinetic and potential energy of a gait, and is generally $\gg 1$ for small running robots.

Regarding efficiency in locomotion, it is well known that scale effects enhance friction forces in small dimensions, making the energy losses a remarkable percentage of the total mechanical work. Mass related forces also scale with dimensions, being proportional to cubic body-length $l^3$. As stress forces are proportional to $l^2$, the same relative jumping height in small dimensions can be achieved with a lower rate of specific work.

Reflecting these and similar considerations, jumping rather than walking can be chosen for an effective locomotion in small robots. The reduced dimensions are exploited to pursue long jumps, which can in addition let the robot overcome obstacles and unevenness. In addition, as the airborne phase is predominant respect to the contact phase, friction losses are minimized along one cycle, concurring to maximize gait efficiency.

II. JUMPING IS JUST A ”LONG HOPPING”? 

A long-jumping gait can be considered at first sight as a hopping gait in which the airborne phase is maximized. Despite this apparent similarity, hopping and jumping are as different as trotting is different from galloping. They are characterized by very different rates of specific power, impact energy and contact forces, implying very different design specifications.

One of the most obvious difference is the ground reaction force. While in hopping it is kept in the order of few body weight (BW), in jumping it can goes from about 8 BW in kangaroo rats [5] to 13 BW in bushbabies [6] and to more than 40 BW in jumping robots [12] - the record goes to froghoppers, with more than 400 BW [7]. This implies the need of an actuation able to provide such a force and the use of buffers to cushion the impact when landing. Indeed the choice of the actuation strategy is a crucial matter in locomotion, it is even more critical for autonomous jumping robots where design limitations impose strict limitations on weight and efficiency.

High take-off forces can imply high peak power as the whole kinetic energy has to be provided in the ground-contact phase. Considering a single jump, the energy needed to launch a mass $m$ over a distance $l$ is

$$W_j = \frac{mgl}{2 \sin(2\alpha)}$$  (1)

where $\alpha$ is the take-off angle. Considering that this energy must be provided during a short percentage of the step time, it becomes clear how the peak power to be delivered by the rear legs can make the long jumping gait prohibitive for most of the commercial actuator, even for the animal muscles. It has
been shown that jumping frogs need to deliver a peak power up to seven times higher than the maximum muscle power [11]. Studying how frogs and insects succeed in performing their jumps was an inspiration for the robot design, as described in the next chapter.

Other aspects related to the jumping gait are the small duty factor and the high changes in the center-of-mass potential energy. The duty factor is defined as the ratio between the time the feet are on the ground and the total stride time. A typical duty factor for hopping is about 0.2, i.e. one foot is on the ground for one fifth of the step period, while in long-jumping gait the duty factor can be even ten times smaller. Such a long airborne phase and short ground contact make it difficult to coordinate the reaction with the ground and thus the robot subsequent orientation during the flight phase. In addition, when landing the potential energy stored in the body’s vertical velocity should be buffered in order to preserve gait efficiency and avoid robot rebounding.

These issues that in jumping locomotion are prominent are also present in all kinds of gaits. Stressing these problems and suggesting some solutions is a challenge that provides insight for any kind of locomotion.

III. DESIGN OF THE JUMPING ROBOT

In designing a light autonomous robot, the actuation and energy systems occupy a crucial point that influence the whole robot dimensioning. Our goal was to design a fast and efficient robot able to move in unstructured terrains and overcome obstacles of dimensions similar or greater than the robot body. The design was kept as simple as possible not only due to weight and size limitations, but also to preserve robot reliability. For these reasons we focused on electric actuation, and in particular on small DC motors.

The proposed design was inspired by nature observation, leading to a very simple structure: four legs, with the two rear legs actuated together by a springs and the forelegs completely passive. Only one degree of freedom is necessary for actuating the jump, while another one is used for steering in the airborne phase.

The idea is to build a mechanically stable locomotion, similar to the one used by several species of insects [8] and some robots [9]. When the uncertainties regarding ground composition and orientation become predominant - as they are for small robots moving in unstructured environments - the choice of a fine gait control is no longer motivated and a fast feed-forward algorithm can be much more successful. Intrinsic mechanical stability and simple controls concur to design a low-demanding-high-performing robot.

A. The rear legs and the actuation system

As pointed out earlier, jumping frogs provide the thrust for the jump with a peak power output several times higher than the maximum muscle power. This is made possible by the presence of an inertial click mechanism and an elastic tendon in series with the muscle. While the muscle contacts, the joint is initially locked and the elastic tendon is stretched. As the force increases the lock begins to slide till it is disengaged and the energy stored in the tendons suddenly released. In this way the peak power output is slightly delayed but strongly increased. In accordance with that, measuring the EMG of the frog’s plantaris muscle, it was shown that at half muscle contraction the body was moved by few percents of the final motion [11].

Elastic tendons and a click mechanism was thus used to actuate the robot rear legs [12]. A tiny DC motor actuated a screw and a cursor was moved along it. On the cursor the legs edge was attached by means of a small NIB permanent magnet, whose attraction was opposing to the spring force. As the cursor was actuated and moved in the way to stretch the springs, the elastic force was increased till overwhelming the magnetic attraction, disengaging in this way the legs (fig.1).

![Fig. 1. The prototype of the jumping robot Grillo. The rear legs are actuated by loaded springs that are released by a passive click mechanism. Here the passive forelegs cannot actively store the impact energy, which is in this case dissipated](image)

Despite the prototype was able to perform long jumps as expected, the jumping frequency was too low to engage a continuous gait, due to mechanical constraints imposed by the cursor motion.

In order to reach a target frequency of 2 Hz a different approach was necessary. An eccentric cam was introduced to load the springs and substitute the click mechanism.

![Fig. 2. The eccentric cam used by Leonardo da Vinci to move the hammer has exactly the same principle of our robot: rotating the cam, the user store energy in the hammer increasing its potential energy. As the escapement position is reached, this energy is suddenly released with an high peak power](image)

The use of escapement mechanisms is well known in
mechanics since centuries (fig.2) and have been widely adopted in the fabrication of mechanical clocks. Nevertheless, these have been poorly adopted in robotics due to the strong mechanical constraints that they introduce. Actually, escapement mechanisms can be feasibly adopted in robots locomotion where cyclic movements are needed, and often generated in feed-forward. Regarding the mechanical constraints in our case, despite the leg elongation is stiffly determined by cam design, the actuation timing can still be regulated by controlling the cam rotation.

The adopted solution is shown in fig.3. An eccentric cam loads a torsional spring that actuates the rear legs. Using a pin joint instead of a sliding cursor helps reducing friction that in small dimensions has a big impact on performances.

Fig. 3. The rear legs are actuated by means of an eccentric cam which load a torsional spring. Actuating in this way the robot it was possible to reduce the weight and the power consumption by a great extent

B. The role of passive forelegs and steering mechanisms

Regarding the passive forelegs, springs are used to store and release the impact energy at a frequency suited for the jumping gait, increasing in this way the efficiency of the robot. This is particularly significative in jumping, where the robot potential energy highly changes during the gait. The mechanical work needed to perform the jump as written in equation (1) contains a percentage of potential energy that depends upon the take-off angle \( \alpha \). As a matter of facts, the ratio between the potential energy \( W_g \) and legs work \( W_j \) during a jump is defined as

\[
\frac{W_g}{W_j} = \sin^2(\alpha)
\]

This means that for a general take-off angle of \( \pi/4 \), half of the energy is used to vertically move the robot and, at landing, this energy should not be lost in the impact. From equation (2) it follows that the gait efficiency can be increased up to 50% using passive forelegs.

At landing, the impact energy can be stored in two ways: in the passive spring recoil at a frequency matching the land-and-take-off timing [12] or in an active energy buffer. The latter implementation was adopted, using the forelegs passive rotation to further rotate the cam, by means of a lever arm and two free-wheel bearings. These are needed to decouple the motor from the cam axis during the forelegs contribution, exactly as it happens when biking downward a slope.

In this way the impact energy is stored in the rear-legs spring and can be used independently of the forelegs recoil. In addition, the forelegs can be used to rotate the cam over the escapement point in order to trigger the gait, implementing an elementary reactive control defined by the mechanical structure.

Regarding the steering capabilities, the very small duty factor makes it prohibitive to actively control the robot flight orientation by coordinating take-off forces. As for the jump thrust, a stabilizing actuation during the contact phase would require high power due to high impact forces and short contact time. Instead, aerial stabilization can be used more efficiently, exploiting air friction or inertial forces. Despite the latter have been successfully used to stabilize hopping robots [13], we preferred the former solution to preserve the robot simplicity. Thus steering is achieved in the airborne phase through maneuvering small aerial appendages, whose function is also to stabilize the body pitch rotation. Due to the relatively small Reynolds number of the robot (Re≈3000) the dimensioning of the small appendages was made by trial and error, without expecting a significant lift from the small wings.

IV. DIMENSIONING LEGS PARAMETERS: COMPLIANCE AND ELONGATION

Dimensioning the robot legs implies dimensioning the springs parameters such as elongation and stiffness. Our intention is not to provide a fine robot dimensioning in a particular environment condition, but to find design relations for ensuring mean jumping performances in unstructured and very different terrains such as concrete, soil or vegetation. In order to point out a reference dimensioning, some simplifications are needed. We suppose that the feet exert coulomb friction with the ground, which is ideally inelastic and that all the losses in the robot are speed-dependant. These hypothesis, which could seem restrictive in modeling robot dynamics, proved to be acceptable in determining parametric relations.

A. The hind legs dimensioning

The rear legs were modeled as a mass-spring-damper system. For simplicity we will consider a compressive linear spring instead of the torsional spring that was used in real prototype. In any case, switching from torsional to compressive spring is quite straightforward imposing relatively small rotations (≤ 20 deg) and the same force pattern on the foot.

At take off, the compressive spring of stiffness \( k \) with equilibrium position in \( x_{eq} = 0 \) elongate from \( x_2 \) to \( x_1 \) with \( x_2 < x_1 < 0 \), accelerating the robot of mass \( m_b \) to a speed \( v_b \). The variables that define the rear legs are \( k, x_1, x_2 \) relative to the spring. The first relation which can be written is the energy balance. The spring should store enough energy to let the robot jump at take-off with a speed \( v_b \)

\[
\frac{1}{2} m_b v_b^2 + m_b g \Delta h_b = \frac{1}{2} k (x_2^2 - x_1^2) - E_d
\]

where the term \( E_d \) represents energy losses and \( \Delta h_b \) is the change in height of the robot center of mass (cm) during leg extension. The energy constraint strictly correlate the leg...
elongation $\Delta x = x_2 - x_1$ and preload to the needed spring stiffness, influencing the contact peak force. This has to be limited to a maximum value to avoid too high foot pressure during the thrust. As a matter of facts, despite the lightness of the robot, high stresses are generated at the rear feet due to the high force exerted on a small contact area. In unstructured environments, such as soil or vegetation, this limit can be quite restrictive. Thus we can write

$$k \cdot x_2 \leq F_{\text{max}}$$

(4)

where $F_{\text{max}}$ depends on feet dimensions, and for our 15-grams robot was limited to 7 N.

Another constraint to be considered is the maximum leg elongation. This is mainly due to the room needed in designing the leg, limited by the robot maximum dimensions

$$|x_2 - x_1| \leq \Delta x_{\text{max}}$$

(5)

With these relations, we have a set of one equation and two inequalities from which an optimization can be run, searching optimal values for the three variables $k, x_1, x_2$.

Despite the existence of such optimum is not proven, we expect to find an optimal combination of the these variables due to the following considerations. Let’s take into account for example the stiffness parameter, with the constraint on the energy to be stored, and inequalities (4) and (5) defining the quantities to be minimized. Using a too stiff spring would reduce the leg elongation, satisfying the constraint in (5) but it would cause a too high peak force. On the other hand, a too low stiffness would require a too long leg elongation an thus a long contact time. Similar considerations can be pointed out on leg elongation and preload, suggesting that values can be found that optimally respect the set of constraints. In the optimization, contact time instead of leg elongation was used in order to take into account jumping performances.

Thus a quadratic optimization was used to minimize the cost factors contact time and maximum exerted force. For consistency, the two quantities were introduced through two scalar factors:

$$\zeta_c = \frac{F_{\text{max},c}}{mg}$$

(6)

$$\tau_c = \frac{T_c}{T_f}$$

(7)

where the maximum thrust force was adimensioned by the robot weight and the contact time $T_c$ by the flight time $T_f$. Thus, the cost function was defined as

$$J = \sqrt{\psi_1 \cdot \zeta_c^2 + \psi_2 \cdot \tau_c^2}$$

(8)

where $\psi_1$ and $\psi_2$ are weighting factors.

Table I summarize the values obtained for the jumping robot. The gait velocity is the take-off speed averaged by contact time and air-friction losses, which reduces the actual speed by about 12%.

### Table I

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
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<tbody>
<tr>
<td>robot mass $m$</td>
<td>0.015</td>
<td>[kg]</td>
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<tr>
<td>take-off speed $v$</td>
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<td>[m/s]</td>
</tr>
<tr>
<td>take-off angle $\alpha$</td>
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<td>[rad]</td>
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<tr>
<td>spring stiffness $k$</td>
<td>0.5</td>
<td>[N/mm]</td>
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<td>initial position $x_1$</td>
<td>3.3</td>
<td>[mm]</td>
</tr>
<tr>
<td>final position $x_2$</td>
<td>13.8</td>
<td>[mm]</td>
</tr>
<tr>
<td>leg elongation $\Delta x$</td>
<td>10.5</td>
<td>[mm]</td>
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</tbody>
</table>

### B. The forelegs dimensioning

Similar considerations were done to determine a reference dimensioning for the forelegs. The impact energy available at landing is

$$E_{\text{land}} = \frac{1}{2} m_b v_b^2 \sin(\alpha)^2$$

(9)

where $\delta \in [0, 1]$ takes into account air friction losses. In order to avoid rebounding, it should be possible to store all of this energy in the forelegs rotation.

In this design, the impact energy is stored and split between the forelegs recoil and the cam rotation. The forelegs can be modeled as a torsional spring of stiffness $k_f$, rotating during compression from angle $\phi_1$ to $\phi_2$ and back during the passive recoil. For simplicity we choose $\phi_1$ corresponding to the spring equilibrium position. Given that the cam torque $M_m$ is about constant, as described in the next chapter, an energy balance can written as

$$E_{\text{land}} = A_s \frac{1}{2} k_f \Delta \phi^2 + (1 - A_s) M_m \Delta \theta + E_d$$

(10)

where $A_s$ indicates the percentage of energy stored in the spring, $E_d$ are the energy losses and $\Delta \theta$ the cam rotation accomplished at the impact. The coefficient $A_s$ is very important in determining how much the forelegs passive recoil influence the subsequent jump. An high $A_s$ implies not only that the energy provided by the forelegs is comparable with the one of rear legs, but also that this energy would highly depend upon ground conditions, making the jumping gait more sensitive to external uncertainties. Choosing a small $A_s$, most of the impact energy would be stored directly in the rear legs, making the jumping gait more robust. The connection between the forelegs and the cam was performed by a four-bar linkage, with a leverage arm defined by the ratio $\Delta \theta/\Delta \phi$.

### V. CAM DESIGN AND JUMPING PERFORMANCES

The cam was designed in order to minimize the room occupied and to keep an almost constant motor torque. As the leg elongation is the bottleneck for limiting the cam dimensions, having a small initial radius is necessary for respecting room limitations. This results in a relatively high radius variation and high pressure angle, which makes not
trivial to operate at a constant torque during the whole cam rotation.

The cam equation can be defined in polar coordinates as

$$\rho = f(\theta)$$  \hspace{1cm} (11)

for $\theta \in [0, 2\pi]$ with $\rho$ varying from $\rho_i$ to $\rho_f > \rho_i$. The difference $\Delta \rho = \rho_f - \rho_i$ define the leg elongation. In order to keep the torque constant the spring is loaded going from $\rho_f$ to $\rho_i$, with higher elongation forces exerted at smaller radius. This obliges us to choose a traction spring with the cam configuration as shown in figure 4, with the cursor sliding in the inner part of the cam.

The cam profile defined as in equation (11) can be determined by imposing the condition on the motor torque to be constant. Figure 5 shows the forces acting on the cursor. The suffix $c$ indicates the forces exchanged with the cam, which determines the motor torque $M_m$, while the suffix $s$ designates the reaction forces due to the spring torque $M_s$. For simplicity, we will neglect the cursor dimensions, considering the forces acting on the same point. The pressure angle $\alpha$ is defined by the cam profile as

$$\tan(\alpha) = \frac{1}{\rho} \frac{d\rho}{d\theta}$$  \hspace{1cm} (12)

As the cursor is not moving radially but in a direction angled of $\gamma$ respect to radius $\rho$, the actual pressure angle is increased by $(\alpha + \gamma) - \frac{\pi}{2}$. This will be taken into account when designing the cam profile, limiting the pressure angle to about 20 deg.

The force balance can be written along two directions parallel and perpendicular to the radius, together with the coulomb-friction equation and the spring action

$$\begin{cases}
\sum F_N = 0 \\
\sum F_T = 0 \\
F_Tc = \eta F_Nc \\
F Ts = \frac{M_s}{T}
\end{cases}$$  \hspace{1cm} (13)

where $\eta$ is the friction dynamic coefficient and $F Ts$ is the force due to the spring torque. Solving the system in (13) it is possible to express the four forces as function of the spring torque and the geometric construction.

As the motor torque is given by

$$M_m = [F_Nc \sin(\alpha) + F_Tc \cos(\alpha)] \cdot \rho$$  \hspace{1cm} (14)

solving (13) $M_m$ can be expressed as

$$M_m = \frac{M_s}{l} \left( \frac{\sin(\alpha) + \eta \cos(\alpha)}{\cos(\alpha + \delta) - \eta \sin(\alpha + \delta)} \right)$$  \hspace{1cm} (15)

where $\delta = \varphi + \gamma - \frac{\pi}{2}$. Geometric relations can be found to relate $\varphi$ and $\gamma$ to cam angle $\theta$ through the lever and radius lengths, while $\alpha$ is defined by equation (12). Choosing the spring parameters as shown in table II, it is possible to numerically find an optimal solution for equation (15) that keep the motor torque constant.

Expressing the cam profile as

$$\rho = A + B\theta + C\theta^2 + D\theta^3 + E\theta^4$$  \hspace{1cm} (16)

the set of coefficients $A, B, C, D, E$ can be obtained as minimizing the torque ripple. In figure 6 it is plotted the motor torque needed, with the parameters chosen as in table II. The great difference in motor and load torque is due to the lever mechanism: while the cam performs an almost complete turn, the leg rotates of about 20 deg.

This design permit to choose a very small motor to empower the robot locomotion. With a 2.3-grams DC motor, providing 0.2 W it is possible to built a robot within a total

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**TABLE II**

<table>
<thead>
<tr>
<th>Parameters</th>
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<th>Units</th>
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<td>stiffness $k_c$</td>
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<td>Nmm/rad</td>
</tr>
<tr>
<td>initial angle $\gamma_1$</td>
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<td>[rad]</td>
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<tr>
<td>final angle $\gamma_2$</td>
<td>0.46</td>
<td>[rad]</td>
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</table>
The robot's rear legs are actuated by a spring loaded during the whole jump by a small motor. At take-off, an escapement mechanism lets the spring release the stored energy, generating a peak of power several times higher than the motor one. On the other hand, passive compliant forelegs cushion the ground contact and partially store the impact energy to empower the subsequent jump.

In order to let the robot jump with high frequency, the actuation system was driven by an eccentric cam. In this way the robot gait can be controlled in feed-forward, simplifying the robot structure and making the gait more efficient and reliable.

The resulting robot weighs about 15 grams and is expected to be able to run at about 1.5 m/s, which would correspond to 30 body length per second. Due to its efficiency, a commercial battery could make the robot run for more than two hours, letting it cover a distance of several kilometers.

**REFERENCES**


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**VI. CONCLUSIONS**

This paper proposes a feasible design for a long-jumping mini robot. The aim is to develop a platform able to locomote with high efficiency in unstructured terrains. This robot is also intended to model the influence of scale effects on legged locomotion. Comparing different gait strategies adopted in nature by different sized animals, it can be inferred that in small dimensions interested by high Froude numbers \(F_r = \frac{v^2}{gl}\), jumping is more effective than walking or hopping.

The robot we propose has a very simple structure: two degrees of freedom for jump powering and steering, passive forelegs and a feed-forward control. The robot’s rear legs