BASIC SOLID MECHANICS FOR TACTILE SENSING

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ABSTRACT

In order to stably grasp objects without using object models, tactile feedback from the fingers is sometimes necessary. This feedback can be used to adjust grasping forces to prevent a part from slipping from a hand. If the angle of force at the object finger contact can be determined, slip can be prevented by the proper adjustment of finger forces. Another important tactile sensing task is finding the edges and corners of an object, since they are usually feasible grasping locations.

This paper describes how this information can be extracted from the finger-object contact using strain sensors beneath a compliant skin. For determining contact forces, strain measurements are easier to use than the surface deformation profile. The finger is modelled as an infinite linear elastic half plane to predict the measured strain for several contact types and forces. The number of sensors required is less than has been proposed for other tactile recognition tasks.

A rough upper bound on sensor density requirements for a specific depth is presented that is based on the frequency response of the elastic medium. The effects of different sensor stiffnesses on sensor performance are discussed.

1.0 INTRODUCTION

Much of the recent work in tactile sensing has been devoted to recognizing objects and features. One method is to obtain an image-like array of an object profile using high density tactile sensors [10,16]. This is useful for identifying the location, orientation and shape of an object with complicated surfaces, or identifying surface defects. Another approach to the recognition problem uses local low level tactile information and object models to recognize objects [7].

There are some low level tactile sensing operations that are useful for basic grasping, where the intent is keeping an object stably grasped in a hand, rather than recognizing it. The requirements for grasping polyhedra on a plane with two fingers without object models have been described elsewhere [5]. The most useful parameters to know are the surface normals, the angle and magnitude of a force at a contact, and whether the finger is touching a corner or edge. These parameters are a subset of those required to recognize general features.

This paper attempts to show how grasping information can be recovered using simplified solid mechanics models and basic contact theory. Consideration of the mechanics has only been done rarely in the design of tactile sensors [11]. There has been some work on sensing forces at fingers using tactile sensation to prevent slip [26]. Other techniques have relied on using rollers to detect the slip of an object in parallel jaw grippers [13]. At a more advanced level, the tactile array approach will provide some useful information for manipulation, such as finding a specific feature that is crucial to orienting a part accurately.

A finger must have a compliant covering to take advantage of the increased prehension stability possible at corners [5]. Another advantage of a soft skin is that contact areas are large enough to distinguish between features. For example, with a very hard skin, an edge and a side will contact with the finger at only a few points, and so may be indistinguishable without finger motion. To distinguish between them, it will be necessary to get either the displacement profile or the contact stresses in a compliant finger covering.

There are two different approaches to the tactile transduction problem. The first approach is based on measuring the deflection of a flexible membrane when it contacts a rigid object, or the height of pegs touching an object [18]. The measurement is sometimes done by an optical sensing scheme [17]. The second approach is based on forces beneath the surface changing electrical contact areas [10], or compressing some resistive material without changing the electrical contacts.

We are interested in determining the actual contact stresses rather than just the contact profile for several reasons. First, although the profile aids in recognition of an object shape, it is only indirectly useful for determining the grasping forces. Another problem is the indirect relation between the shape of the object and the profile of deformation, especially when there are both normal and tangential forces applied at the contact. Because the compliant material is not perfectly compressible, it will tend to pile up outside the contact region in a complicated manner.

A mathematical problem with treating the contact as a deformation is that superposition does not hold. In general, forces will superimpose, but displacements will not. For example, the stresses due to two points close together each indenting 1 mm. into an elastic medium are not equivalent to the sum of the stresses from each point indenting 1 mm. by itself. However, if we deal with the contact between a rigid object and a compliant skin in terms of stresses, superposition will hold.

Consider a grain of sand pressed between a finger and a smooth surface. The profile change due to the height of the grain would be very small, but there would be high stress concentrations near its edges that should be readily detectable. One researcher notes that a ridge of less than 20 μm height is perceptible for humans [12].

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2.0 USING STRAIN FOR TACTILE SENSING

When determining contact stresses, there are practical reasons for measuring these stresses beneath the elastic material, rather than on the surface. Since we may be interested in both normal and tangential (friction) forces at the contact, we would need two different types of sensor at the surface. When sharp rigid objects indent an elastic material, very high stresses are developed at the surface. These stresses are reduced by distance from the application area, so a fragile sensor would be better protected beneath a layer of skin. We note that biological tactile sensors are beneath the surface of the skin. A rather complete analytical and experimental study of stresses beneath the skin and the mechanoreceptor responses to contacts without tangential forces was done by Phillips and Johnson [19].

The general three dimensional analysis of stress and strain leads to very complicated expressions. If contact forces are constant along a finger, a two dimensional analysis will describe the behavior in a slice perpendicular to the finger axis. In the remainder of this paper we analyze line, edge, and plane contacts for which a 2 dimensional analysis is exact for infinitely long contacts, such as a line load, and approximate for short contact lengths. We will speak of a point indenter or point force while meaning a planar section of a line contact.

We will now examine the 2 dimensional behavior of an infinite homogeneous elastic medium for the simplest contact case, a point force with negligible contact area. To find the stresses within the elastic half plane, we will first find the stress distribution due to a concentrated normal force at the surface, as in Figure 1. The internal stresses due to a general contact stress on the surface can be found from the superposition principle.

The analysis from Timoshenko [27] shows that the internal stresses have a "simple radial distribution". That is, all the stress is in the direction of a radial line from the point of application of the force.

From Timoshenko [27], for the concentrated normal force we get

$$\sigma_r = \frac{-2P}{\pi}\,\frac{\cos\theta}{r}$$

$$\sigma_\theta = 0$$

$$\tau_{r\theta} = 0 \tag{1}$$

where σ_r is the radial stress at (r, θ), σ_θ is the stress in the plane at (r, θ) normal to the radial stress, $\tau_{r\theta}$ is the shearing stress in the r,θ plane, P is the force per length, and r is the distance from the point of application.

For a force inclined from the vertical by the angle α,

$$\sigma_r = \frac{-2P}{\pi r}\cos(\alpha+\theta)$$

$$\sigma_\theta = 0$$

$$\tau_{r\theta} = 0 \tag{2}$$

To get stresses in cartesian coordinates, we apply the tensor transformation:

$$\sigma_x = \sigma_r \cos^2\theta$$

$$\sigma_y = \sigma_r \sin^2\theta$$

$$\tau_{xy} = \sigma_r \sin\theta\cos\theta \tag{3}$$

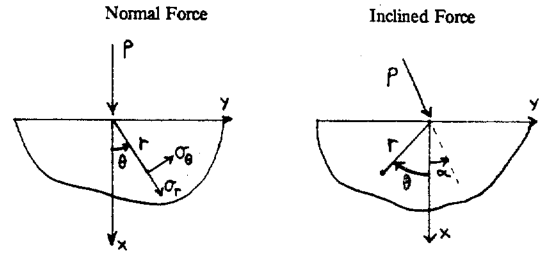

Figure 1. Point Forces on Elastic Half Plane

So finally, the internal stress due to a point force is:

$$\sigma_x = \frac{-2P}{\pi r}\cos(\alpha+\theta)\cos^2\theta$$

$$\sigma_y = \frac{-2P}{\pi r}\cos(\alpha+\theta)\sin^2\theta$$

$$\tau_{xy} = \frac{-2P}{\pi r}\cos(\alpha+\theta)\sin\theta\cos\theta \tag{4}$$

where

$$r = \sqrt{x^2 + y^2}$$

$$\cos\theta = \frac{x}{r}, \quad \sin\theta = \frac{y}{r}, \qquad \cos(\alpha+\theta) = \frac{x}{r}\cos\alpha - \frac{y}{r}\sin\alpha$$

All pressure sensors have an output based on the strain of the sensor, which is the fractional change in the linear dimensions of a small cubic volume element. This volume change is to first order independent of the shear stresses, which change the angles of the faces in the cube, and not the dimensions. If we assume a linear elastic medium, we can apply Hooke's Law:

$$\varepsilon_x = \frac{\sigma_x}{E} \tag{5}$$

where E is the modulus of elasticity (N/m^2), and ε_x is the strain along the x axis.

For materials that are not completely compressible, a contraction along one dimension will be coupled with expansions along the other two. For an incompressible cube which undergoes a strain of 1% along one axis, there must be corresponding strains in the opposite sense of about .5% along the other two axes to maintain constant volume. The ratio of these two strains is called Poisson's ratio, and characterizes the compressibility of the material. For a completely compressible material, Poisson's ratio is 0. For a medium that does not change in volume with compression, such as a water filled sack or rubber, Poisson's ratio can be taken as 0.5. Since rubber-like materials are popular for covering robot fingers and tactile sensors this assumption will be used.

The relations between the stresses and strains are [27]:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y+\sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x+\sigma_z)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x+\sigma_y)\right] \tag{6}$$

where ν is Poisson's ratio=.5, and ε is the strain along one dimension.

There are two simplifying assumptions that can be applied to elasticity problems with the appropriate symmetry. For an infinite line force on an elastic half space, the strain in the direction of the line must be zero by symmetry. This is the plane strain assumption. Consider a slice of unit width out of the elastic half space, with a line load acting on it. On the faces of this slice, the stresses normal to face must be zero to satisfy the boundary conditions. This is the plane stress assumption.

If a stress pattern is essentially constant along an axis, a cross section of the stress and strain can be analyzed in just a half plane instead of a half space. For this analysis, we will assume a state of plane strain; there is no strain in the z direction. This assumption will be reasonable if the contacts are long compared to the finger radius. Phillips and Johnson [19] found that an assumption of plane stress had a better qualitative agreement with the response of mechanoreceptors in finger skin, but the plane strain assumption will be used here because the contacts are assumed to be infinitely long. The form of the strains under these two assumptions is similar.

So from the plane strain assumption $\varepsilon_z = 0$,

$$\sigma_z = \frac{1}{2}(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \frac{1}{2}\left[\sigma_y + \frac{\sigma_x+\sigma_y}{2}\right]\right] = \frac{3}{4E}(\sigma_x - \sigma_y)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \frac{1}{2}\left[\sigma_x + \frac{\sigma_x+\sigma_y}{2}\right]\right] = \frac{3}{4E}(\sigma_y - \sigma_x) \quad (7)$$

For the stresses due to the point indentor or line load (4), the strains from (7) become:

$$\varepsilon_x = \frac{3}{4E}\left[\frac{-2P}{\pi r}\cos(\alpha+\theta)\right]\left[\cos^2\theta - \sin^2\theta\right]$$

$$= \frac{-3P}{2\pi E r^4}\left[x\cos\alpha - y\sin\alpha\right]\left[x^2 - y^2\right]$$

for $\alpha = 0$

$$\varepsilon_x = \frac{-3P}{2\pi E r^4}\, x\,(x^2 - y^2) \quad (8)$$

Figures 2 and 3 show the strains in the homogeneous elastic medium for a line load normal to the surface and one inclined 30 degrees.

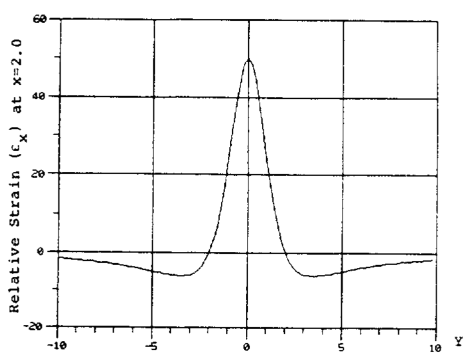

Figure 2. Strain for Line Load Normal to Surface

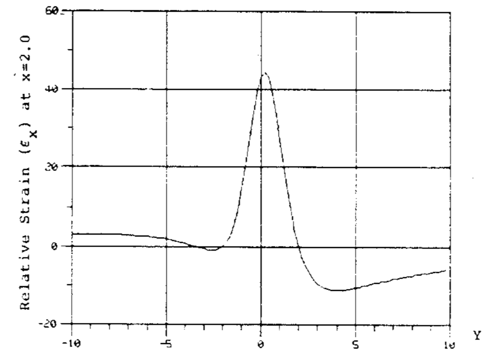

Figure 3. Strain for Line Load Inclined 30 Degrees

2.1 Measuring Strain

Since our plane strain assumption gives a change in volume that depends only on the strain in the x direction, there are some implications for volume dependent strain sensors. Figure 4 shows how a transducer based on the volume resistance could be used to determine the strain at a certain depth. Consider a block of conductive material with the electrodes on the top and bottom. The resistance is given by:

$$R = \frac{\rho L}{A} = \frac{\rho x}{yz} \quad (9)$$

where ρ is the resistivity, R is the resistance, L is the length, and A is the area.

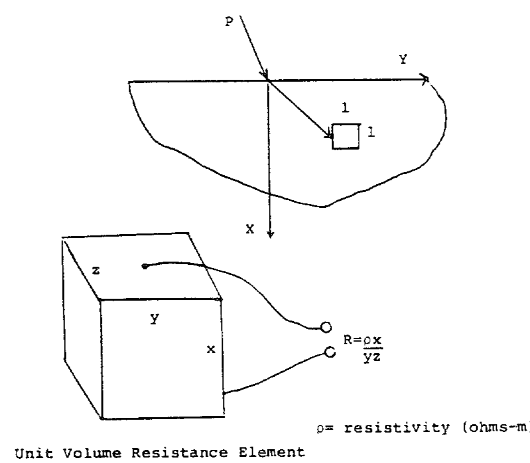

Figure 4. Strain Sensor Based on Resistance

For plane strain in the x direction, the new dimensions are:

$$x' = x(1+\varepsilon_x)$$

$$y' = y(1+\varepsilon_y) = y(1-\varepsilon_x) \text{ since } \varepsilon_y = -\varepsilon_x$$

$$z' = z$$

for a unit volume element

$$R = \frac{\rho(1+\varepsilon_x)}{(1-\varepsilon_x)} \approx \rho(1+\varepsilon_x)(1+\varepsilon_x+\varepsilon_x^2+\cdots) \quad (10)$$

$$\approx \rho(1+2\varepsilon_x)$$

This transducer would output the variable of interest. A similar result can be shown for the plane stress assumption:

$$R = \frac{\rho(1+\varepsilon_x)}{(1+\varepsilon_y)(1+\varepsilon_z)} \approx \rho(1+\varepsilon_x-\varepsilon_y-\varepsilon_z) = \rho(1+2\varepsilon_x) \tag{11}$$

3.0 DETERMINE FORCE ANGLE FOR A LINE LOAD

Can the angle of inclination, location, and magnitude of a line load be recovered from the type of sensor described previously, that measures only one parameter, ε_x? Assuming negligible sensor dimensions, it is straight forward to set up the equations for three strain sensors equally spaced on a horizontal plane beneath the surface. Referring to Figure 5 they are:

$$\varepsilon_{x_1} = \frac{-3P\left[x\cos\alpha - (y+b)\sin\alpha\right]}{2\pi E\left[x^2+(y+b)^2\right]^2}\left[x^2 - (y+b)^2\right]$$

$$\varepsilon_{x_2} = \frac{-3P\left[x\cos\alpha - y\sin\alpha\right]}{2\pi E(x^2+y^2)^2}\left[x^2 - y^2\right] \tag{12}$$

$$\varepsilon_{x_3} = \frac{-3P\left[x\cos\alpha - (y-b)\sin\alpha\right]}{2\pi E\left[x^2+(y-b)^2\right]^2}\left[x^2 - (y-b)^2\right]$$

where x is the sensor depth and b is the sensor spacing.

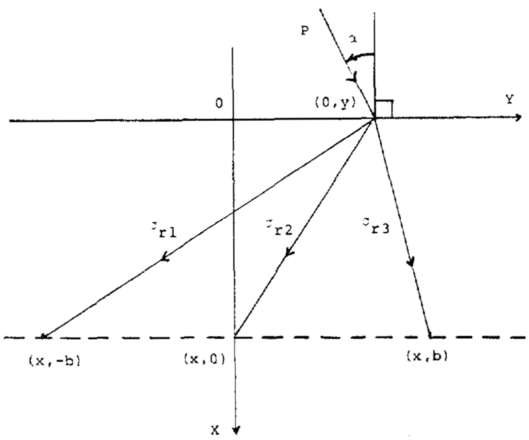

Figure 5. Three Equally Spaced Strain Sensors

We can use "equation counting" methods [22] to determine if these 3 measurements are sufficient to uniquely determine the line load on the surface. The implicit function theorem [25] states that if the system of equations (12) is continuous and has continuous first partial derivatives with respect to the independent variables, and if the Jacobian is non-zero, then there will be a unique and continuous solution to those equations. The determinant of the Jacobian of eq. (12) is not identically zero, but does disappear for some values of parameters.

To determine where the extra solution points are, the system of equations was solved by elimination of the magnitude and angle of the force, to get a polynomial in y and the measured strains:

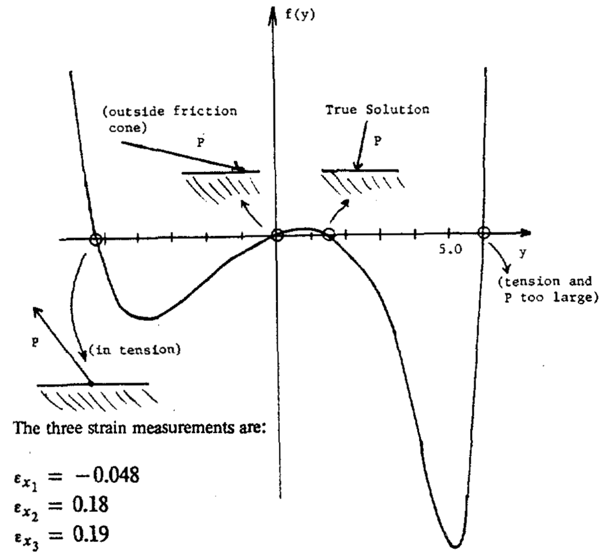

Figure 6. Finding the Force Location with Multiple Roots

$$\begin{aligned} f(y) = {} & 2\varepsilon_{x_2}\left[x^2+y^2\right]^2\left[x^2-(y-b)^2\right]\left[x^2-(y+b)^2\right] \\ & - \varepsilon_{x_1}\left[x^2+(y+b)^2\right]^2\left[x^2-y^2\right]\left[x^2-(y-b)^2\right] \\ & - \varepsilon_{x_3}\left[x^2+(y-b)^2\right]^2\left[x^2-y^2\right]\left[x^2-(y+b)^2\right] = 0 \end{aligned} \tag{13}$$

This eighth degree polynomial has multiple real roots close to the sensor locations. It is interesting to note that if the sensor depth and the spacing are equal (x=b), one of the false roots will always be at y=0. Constraints on the possible forces, such as magnitude ranges and angle limits, allow some of these solutions to be rejected. For instance, a reasonable constraint might be that all forces are directed into the surface (compressive). Tension will not occur at the contact without an adhesive bond. If there are still possible roots left, a fourth sensor measurement will be necessary to get a unique solution. Figure 6 shows an example with false roots that can be discarded.

4.0 DISTINGUISH A VERTEX FROM A SIDE

In this section, we will try to differentiate between the strains beneath the skin due to the vertex and the side contacts. Figure 7 shows the assumptions used for these contacts. Assume the side is perpendicular to the finger and is moving perpendicularly into the skin. For the vertex, assume its center line is perpendicular to the edge of the finger and is moving perpendicularly into the skin. These can be viewed as contacts with a normal force but no moment (Figure 7a).

Figure 7b shows normal contacts with a tangential force. The most general case of a moment and a traction force is shown in Figure 7c. Restricting the analysis to the 2 dimensional case with plane strain, we will attempt to characterize the strains that would be measured for normal contacts without a tangential force.

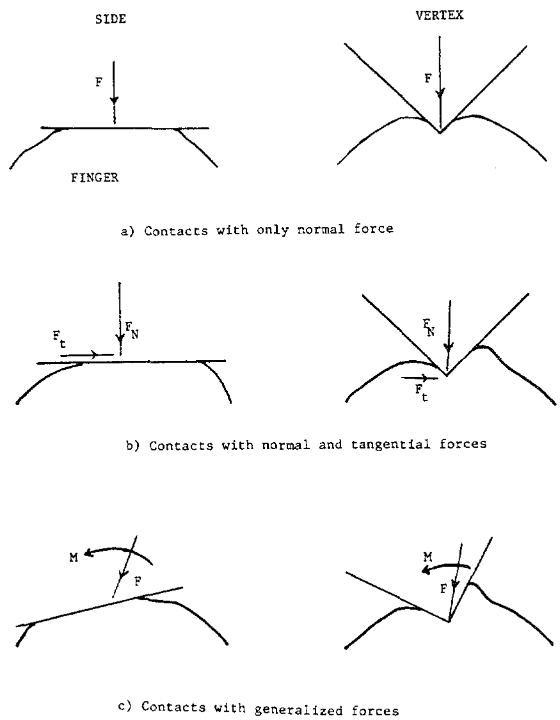

Figure 7. Finger Contacts with Side and Vertex

For this analysis, we will ignore traction from friction while indenting normally into the elastic medium. With friction present, the elastic medium would adhere to the object and cause surface shear stresses as it tried to flow away from the indentation. The normal stresses are not affected very much by this friction [4].

What does a vertex feel like? In this part, we will consider the vertexes touching the finger as in Figure 8. The first case appears to be the right side of a rectangular indenter. If we could consider the finger and the contacting side extending far away from the right side, then the left side of the indenter would have no effect on the right side. The surface stress for a rectangular indenter on an elastic half plane is given by [3]:

$$\sigma_x(x = 0) = \begin{cases} \dfrac{P}{\pi\sqrt{a^2-y^2}} & \text{for } |y| < a \\ 0 & \text{otherwise} \end{cases} \qquad (14)$$

where P is the force per unit length, and a is the half width of the contact.

The second case is roughly equivalent to a wedge of 90 degrees being pressed into an elastic half plane. This stress is given by [8,23] as:

$$\sigma_x(x = 0) = \begin{cases} K cot\dfrac{\theta}{2}\cosh^{-1}\dfrac{a}{|y|} & \text{for } |y| < a \\ 0 & \text{otherwise} \end{cases} \qquad (15)$$

where K is a function of Poisson's ratio and the modulus of elasticity, and θ is the vertex angle.

Figure 9 shows the stresses and strains for these 2 different vertex contacts. A vertex looks very much like a line load especially for small indentations and small vertex angles. If the line load and vertex have approximately the same strain profiles, the method of section 3.0 could be used to find the angle of force at this contact.

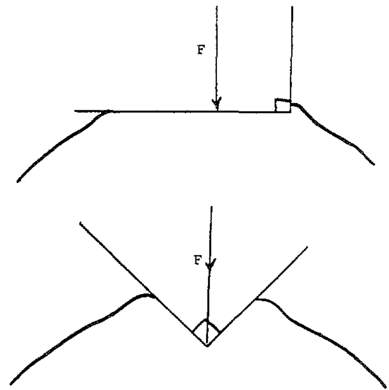

Figure 8. Two Extreme Vertex Cases

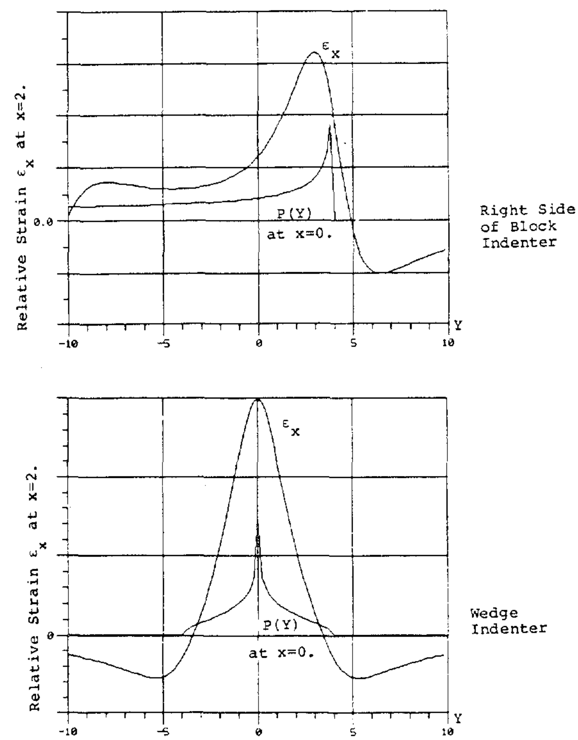

Figure 9. Stresses and Strains for Vertexes

We shall now consider the contact of a stiff straight side and a circular elastic finger as in Figure 10a. This two dimensional problem is still too difficult to analyze analytically, so we will apply Saint-Venant's Principle, as explained by Timoshenko [27]:

This principle states that if the forces acting on a small portion of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, this redistribution of loading produces substantial changes in the stresses locally but has a negligible effect on the stresses at distances which are large in comparison with the linear dimensions of the surface on which the forces are changed.

If the area of contact is small compared to the finger radius, we can approximate the area near the contact as an infinite elastic half plane. This approximation will be most accurate near the outer surface of the finger in the quadrant where the contact is. It is also necessary to reformulate the contact stresses for the cylindrical finger to match the half plane approximation.

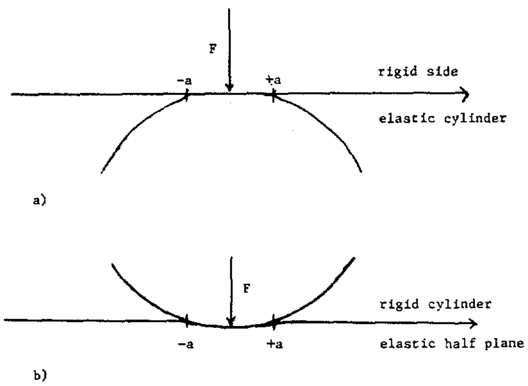

Figure 10. Approximating a Contact with a Side

In the solid mechanics literature, the common problem is the rigid cylinder indenting an elastic layer, the punch problem, rather than an elastic cylinder contacting a rigid plane. Bentall and Johnson [2] have done an analysis where the elasticity of the cylinder and layer are equal. Hahn and Levinson [9] have analyzed the stresses for the contact of a rigid cylinder and a cylinder covered with an elastic layer. That analysis is probably most relevant to the rigid finger with an elastic covering contacting a rigid surface, but as in a lot of the contact literature, the form of the solutions (infinite series) are inconvenient to work with.

For a rigid cylinder indenting an elastic half plane, as in Figure 10b, the stress on the surface is given by Conway [3] as:

$$\sigma_x(y) = \frac{2P}{\pi a^2}\sqrt{a^2-y^2} \tag{16}$$

where a is the half width of the contact region and P is the force per unit length of the cylinder. The stress and strain for this contact are shown in Figure 11.

The half width of the contact from [2] is:

$$a = 2\left[\frac{4RP(1-\nu^2)}{\pi E}\right]^{\frac{1}{2}} \tag{17}$$

where R is the radius of the cylinder.

For equilibrium in the contact region, the stresses should be equal and of opposite sign on the cylinder and the surface. Now for an elastic cylindrical finger contacting a side, there should be no infinite stresses such as occur at sharp edges of indentations. Also, the stress will have a peak value in the center of the contact, and approach zero at the edges. Using this justification, we will assume that the stress on the finger can be approximated by eq. (16) for small indentations.

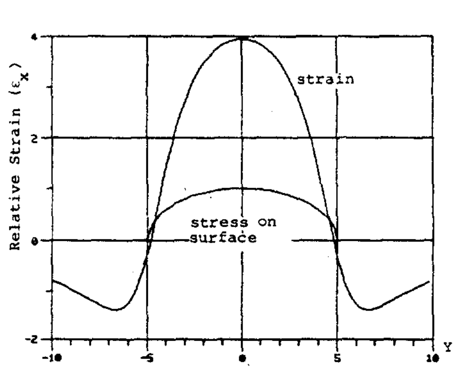

Figure 11. Stresses and Strains for Cylindrical Indentor

The stresses due to contacting a side and a vertex are distinguishably different. For the side, there is a smooth, continuous stress function. For the vertex, there is a discontinuity in the stress at the tip of the wedge. (Actually, the medium deforms around that discontinuity, spreading the stress over a finite area). It should be easy to distinguish the two just by looking at the width of the strain pattern. For the same elasticity and pressure, the wedge has greater indentation depth with a smaller contact area than the side contact. If the vertex angle is less than 90 degrees, there will be a significant difference between the two contact widths. Arguably, this task would be simpler if the indentation profile were available.

5.0 DETERMINING THE SIDE-FINGER CONTACT FORCE

For a finger touching the side, we will assume that the contact is of the form shown in Figure 7b, with both normal and tangential components, but no moment. The tangential components will be modeled as all being in the same direction with a force at the surface that is directly proportional to the magnitude of the normal force [24]. Figure 12 shows this assumption for a general pressure pattern. (The tangential force must be less than μN, the friction force).

The stress below the surface was found for an elliptical stress distribution on the boundary [24], but that expression is too complicated to use for our purposes. To get a better qualitative understanding, the gross assumption will be made that the normal force is relatively constant over a region, and zero outside that region. Figure 13 shows the rough agreement between the strains due to the pressure distribution of eq. (16) and a constant pressure over the same area. By St. Venant's Principle, if the sensors were even deeper, and the contact area smaller, the difference would be less significant. (A parabolic approximation would be a lot more accurate).

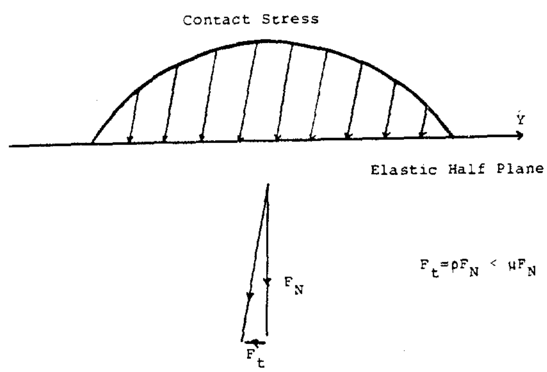

Figure 12. Tangential Stress Proportional to Normal Stress

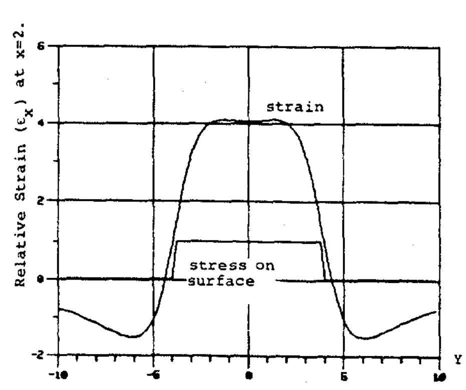

Figure 13. Approximation by a Constant Pressure

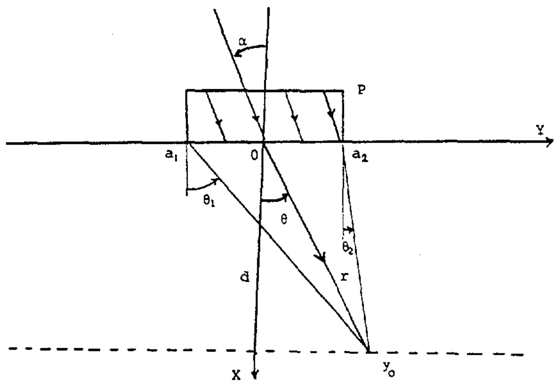

Figure 14. Calculating Strain from Constant Pressure

To find the strain underneath for the constant stress shown in Figure 14, we integrate along the surface where

$$P(y) = \begin{cases} P \text{ for } a_1 < y < a_2 \\ 0 \text{ else} \end{cases}$$

Applying the superposition principle to the strain due to a line contact eq. (8):

$$\varepsilon_x(x,y_o) = \int_{-\infty}^{\infty} \frac{-3\left[x\cos\alpha - y\sin\alpha\right]\left[x^2-y^2\right]P(y_o-y)}{2\pi E r^4}\,dy$$

Substituting the integration limits for $P(y)$ gives:

$$\varepsilon_x(x,y_o) = \frac{-3P}{2\pi E}\int_{y_o-a_2}^{y_o-a_1}\left[\frac{x\cos\alpha - y\sin\alpha}{(x^2+y^2)^2}\right]\left[x^2-y^2\right]dy$$

where $r^2 = x^2+y^2$

Performing the integration gives the strain:

$$\varepsilon_x(x,y_o) = \tag{18}$$

$$\frac{3P}{4\pi E}\left[\begin{array}{l}\left[\dfrac{(y_o-a_1)^2-x^2}{x^2+(y_o-a_1)^2} + \dfrac{x^2-(y_o-a_2)^2}{x^2+(y_o-a_2)^2} + \log\dfrac{x^2+(y_o-a_2)^2}{x^2+(y_o-a_1)^2}\right]\sin\alpha \\ \\ -\,2x\left[\dfrac{y_o-a_1}{x^2+(y_o-a_1)^2} - \dfrac{y_o-a_2}{x^2+(y_o-a_2)^2}\right]\cos\alpha\end{array}\right]$$

The methods of equation counting can also be applied to these strain measurements to determine if 4 measurements are enough to recover P, a_1, a_2, and α.

The more complicated contact with generalized forces of Figure 7c can be treated as the superposition of stresses due to forces plus the stresses due to the moment load. The elasticity literature deals with moments applied to punches indenting an elastic medium [1,6]. In those cases, the pressure distribution due to the moment load is an odd function symmetric about the origin, with half the surface in tension and the other half in compression, so the moment can significantly affect the internal strains.

6.0 SENSOR DENSITY REQUIREMENTS

In section 3, a method to determine the location, direction, and magnitude of a line contact was outlined that required at least three strain sensors. A more general question is, how many strain sensors are needed to identify a general stress pattern on the surface? This problem will be considered in the context of the spatial frequencies of the applied stress. The frequency of the stress pattern is just the number of sinusoidal variations in pressure per unit length. Note that these are not time dependent variations. We shall assume that the pressure pattern is approximately bandlimited, that is, most of the stress spectrum is concentrated at low frequencies. This is the case for contact stresses in the form of (15) and (16).

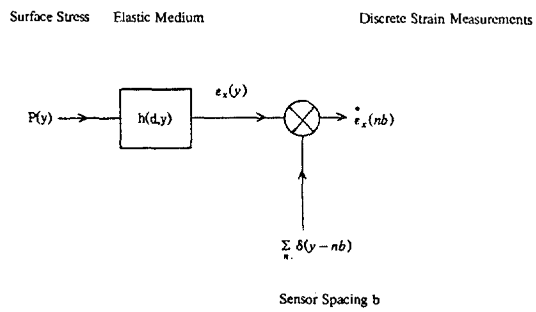

Figure 15. Discrete Strain Measurements

Figure 15 shows a linear system representation of this discrete measurement process, where the sensors are of negligible width. The sampling theorem says that any bandlimited signal can be recovered from discrete samples of it if the samples are taken at a sufficiently high frequency, the Nyquist rate. (For example see [15]). Here the sampling frequency is the minimum strain sensor spacing needed to avoid aliasing when recovering the continuous strain measurements from the discrete samples. If there are not enough sensors, the strain due to the high frequency components will appear as noise when the continuous strain is recovered from the samples by low pass filtering.

Since the strain at a point beneath the surface is a linear function of the two stress components for plane strain, the total strain at a point in the elastic medium can be found by the superposition of the strains underneath due to each stress component on the surface. It should be possible to treat the normal and tangential stresses separately, and determine the highest frequencies of interest for each. If the force on the boundary has no tangential components, the pressure distribution $P(y)$ will be a scalar at each point. The superposition integral is:

$$\varepsilon_x(d,y_o) = \int_{-\infty}^{\infty} \frac{P(y)}{E} h(y_o-y)\,dy \tag{19}$$

where $P(y)$ is the pressure distribution on the surface and $h(y)$ is the strain at depth d due to a unit normal pressure point at the origin. From equation (8):

$$h(y) = \frac{-3\,d(d^2-y^2)}{2\pi\,(d^2+y^2)^2} \tag{20}$$

The spectrum of the "impulse response" $h(y)$ can be found from simple properties of Fourier transforms:

$$\frac{d}{\pi(d^2+y^2)} \longleftrightarrow e^{-d|2\pi s|}$$

$$\frac{d^2}{\pi(d^2+y^2)^2} \longleftrightarrow \pi e^{-d2\pi s}\left[\frac{1}{d2\pi} + s\right] \text{ for } s > 0$$

$$\frac{dy^2}{\pi(d^2+y^2)^2} \longleftrightarrow \pi d e^{-d2\pi s}\left[\frac{1}{d2\pi} - s\right] \text{ for } s > 0$$

$$\frac{-3d(d^2-y^2)}{2\pi(d^2+y^2)^2} \leftrightarrow -3\pi dse^{-d2\pi s} = H(s) \quad (21)$$

$$\text{for } s > 0$$

Here we have neglected the singularities at the origin, because the finger has finite dimensions, and can not have an infinite wavelength pressure distribution.

The frequency response of the elastic medium as shown in Figure 16 is not strictly bandlimited, but it does have very steep skirts. We shall assume that the overall frequency response $P(s)H(s)$ has a high frequency response like $H(s)$, where $P(s)$ is the pressure spectrum.

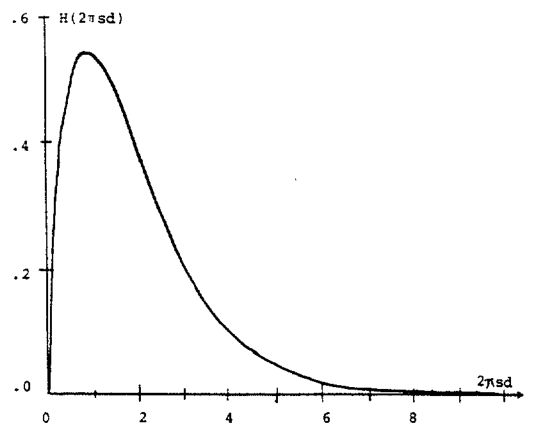

Figure 16. Spatial Frequency Response of Elastic Medium

We can choose a sampling frequency by deciding how much aliasing is permissible. This requires an engineering judgement based on the estimated accuracy of the strain sensors. It seems that any aliasing that was less than one part in 1000 of full scale would be negligible because most sensors would not have a dynamic range much better than that. Also, non-linearities and temperature instabilities get to be of at least the same order of magnitude.

The following rough estimate can be used for sampling requirements. We want the value at the tail to be down a factor of 1000 from the maximum which occurs at $2\pi sd = 1$. So:

$$\frac{e^{-1}}{fe^{-f}} = 10^3, \text{ or } f = 10.2\,. \quad (22)$$

The samples should be at twice f so $2\pi sd \geq 20.4$, or $sd > 3.2$. Here s is the number of samples per unit length.

For a sensor depth of 1.0, having a sensor density of 3.2 sensors per unit distance would allow recovery of the continuous strain response due to a normally directed surface stress distribution with aliasing errors down at least a factor of 1000 from the desired signal. This sampling rate is probably too conservative. The types of features touched may have rounded edges, and start out with a more band limited signal, so a lower sensor density could be adequate.

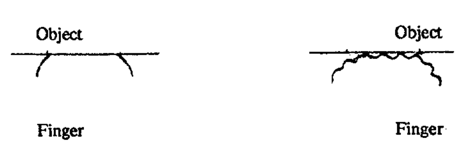

Figure 17. Fingers Without and With Ridges

6.1 Using Finger Ridges to Enhance Sensitivity

The infinite elastic medium has a peak response when the wavelength times the strain sensor depth $= 1/2\pi$. There seems to be an interesting possibility of maximizing the strain sensors' outputs by locating them at the maxima and minima of strain, and predetermining the frequency and phase of the pressure. This could be done in principle by inserting a thin and flexible sinusoidal grating between the finger and the object touched, which could superimpose a sinusoidal stress on the regular stress. Figure 17 gives a simple method of adding the sinusoid of the desired frequency using a ridged finger.

In Figure 18 we compare the strain amplitude between the unridged and ridged fingers, where the ridges are of the optimal wavelength. In both cases the sensors are at the same depth. In the unridged case the total contact stress is about 1.5 times the ridged case. Even neglecting the difference in total contact stress, the ridges can enhance the amplitude by a factor of 2. The sensors should be located beneath and between ridges to detect the maximum amplitude peaks. This technique is similar to the electrical chopper that allows a DC level to pass through an AC coupled circuit. It is curious to note that in the human finger, there is a mechanoreceptor located directly beneath every papillary ridge [12].

Phillips and Johnson [19] gave the depth of this strain measuring mechanoreceptor in the macaque monkey as about 500 μm. This is about the same depth as in the human finger [20]. The optimum ridge spacing for this depth is about 3 mm, much larger than the papillary ridge spacing on the authors' fingers. However, it is interesting to speculate that fingerprints may still have some role in enhancing strain measurements.

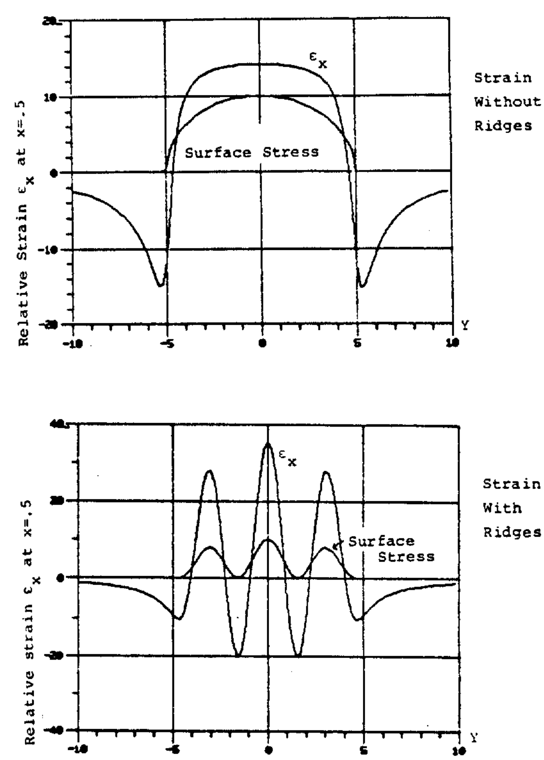

Figure 18. Effect of Ridges on Strain Amplitude

7.0 DISCUSSION

It seems that some of the recognition tasks would be considerably easier using a sensor that responded to deformation. That would make distinguishing a vertex and a side easy work. However, other tasks, such as sensing imminent slip, seem to require the strain sensor approach. Sensors that combine surface shear sensors, depth strain sensors, and surface deflection sensing would simplify the problem considerably.

What effect does the skin compliance have on the tactile sensing abilities? A stiffer skin will develop greater contact stresses for the same indentation, but the subsurface deformations (the measured strain) will be the same [19]. The smaller contact area and deformation with a less compliant material helps to make the approximations we have used more valid. Greater stiffness in the skin may also serve to protect the sensors because a stiffer skin has more resistance to puncture.

A very compliant skin on a sensor may have saturation effects sooner than a harder skin. Figure 19 shows three fingers applied to a surface with the same normal force, but with different covering softness. For the very stiff skin, not enough contact has been made to determine the size of the particle. For the very soft skin, the stress due to the small particle may become proportionally less significant than the stress due to contact with the surface around the particle. One way to choose a skin compliance for good sensing characteristics would be to decide on a typical finger force range and the smallest contact area of interest, and use this information to determine the necessary modulus of elasticity.

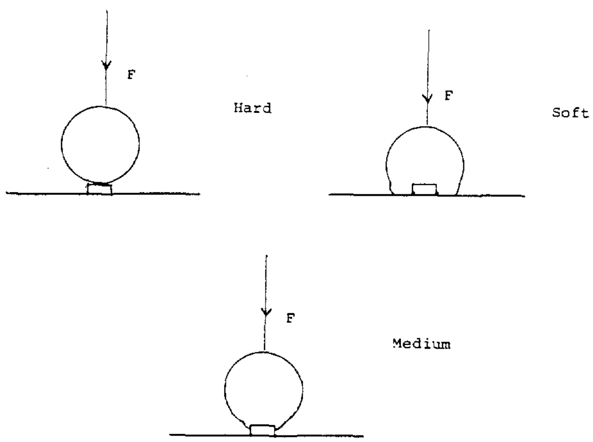

Figure 19. Contacts for Different Skin Compliances

In psychophysical experiments, subjects are instructed that there will either be one or two points indenting their skin. It appears that the sensors should be able to distinguish between these two cases for two-point distances significantly smaller than the individual receptive fields of each sensor. Here, the resolution is not limited by the density of the sensors, but by the measurement accuracy of the sensors.

An area for further research is the spatial bandwidth that is required for feature recognition. A test such as the two point discrimination test would seem to have very high bandwidth requirements because of the two impulse functions, but it may turn out that only the low frequencies are important. The two-point discrimination test [28], seems to be the classic test for tactile sensor resolution. While we are not interested in resolving between two point indenters, this problem may be useful for setting a smaller upper bound on the density of sensors.

A useful way of approaching this problem is to consider each strain sensor as having a response function that overlaps its neighbor. A point force on the surface will give an output for each sensor that is proportional to the height of the response function underneath that point. For two point forces on the surface, the outputs will be the linear superposition of the responses of each point force. We can measure the output in each sensor channel, and determine if it is consistent with just a single point force. However, there might be different combinations of point forces that can give the same output. A very loose analogy could be made to the color matching problem [21].

In Figure 20 there are locations where two point forces can be applied that will give a response in the two sensors equivalent to a single point force. In Figure 21, a third sensor has been added to allow discrimination between all the two point and one point cases, limited by the measurement accuracy. It will be difficult to distinguish points that give responses way down on the "tails" of the channel. If we consider a contact with a vertex as being roughly the same as a point force, and a contact with a side being equivalent to many closely spaced point contacts, this simple method could be sufficient for disambiguating the two cases.

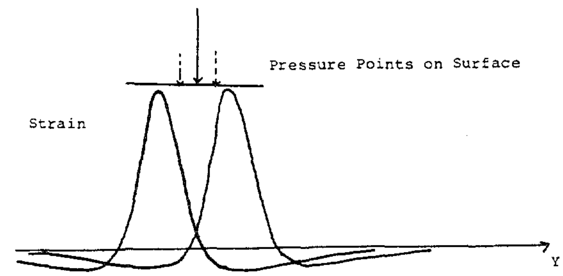

Figure 20. Two Strain Response Sensors

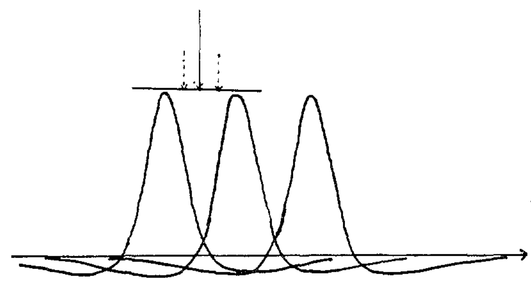

Figure 21. Three Strain Response Sensors

It is probably reasonable not to spread the sensors too far apart. A guess is that the practical maximum separation would be less than twice the depth of the sensors. This would give a two-point discrimination limit about equal to the depth of the sensors. (Note that for the two-point discrimination task, the modulus of elasticity has no effect on the width of the strain response, so it will have no effect on the resolution). More work should be done on optimizing the depth and sensor spacing to optimize the discrimination.

8.0 CONCLUSION

In this paper, we have used a simple linear elastic model to predict strains beneath a compliant skin for a finger touching a knife edge (line load), a corner, and a flat surface. Appropriate approximations were made to get simple expressions for the contact stresses for a cylindrical finger contacting a side. In general, three strain measurements were shown to be necessary and sufficient for determining the location, magnitude, and direction of force for a line contact, but degenerate cases with multiple solutions are possible. Further work needs to be done in recovering the angle of force from a side contact, and the analysis needs to be extended to contacts with other curvatures.

The elastic medium was examined as a signal processing element, which led to the interesting possibility of maximizing strain sensor output by adding a grating to the sensor surface with a period equal to the sensor depth times 2π.

We have not attempted to determine stresses and strains for the generalized three dimensional contact. At least five strain measurements are required to determine the magnitude, two angles, and y,z location of a point force on an elastic half-space. These equations are quite complicated [14]. It may be possible to find this information by combining two orthogonal planar solutions, but this was not attempted here.

The discussion here seems to indicate that four sensors, with fairly high dynamic range, may be adequate for the planar case for finding force magnitudes and directions, and for distinguishing between two contact types (vertex and side). With a thick skin, one can get a lot from a few sensors. This number of sensors will probably not suffice for determining curvature and more complicated force resolving problems.

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