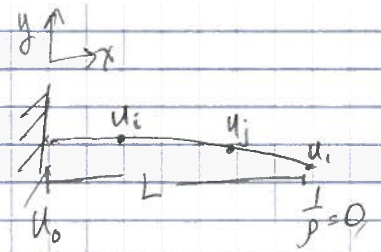
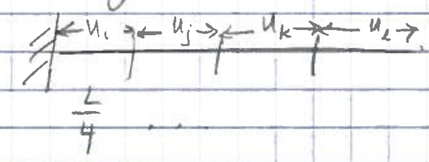


Parametrization in 2D.

$\phi = 0$
$\delta = 0$

$u = \frac{x}{L}$ (assume small δ)

If multiple possible sensor locations are being choosen,



Could restrict to ranges, for optimization

More sensors could allow for higher order approximations.
In this case,
Connect three 3rd order polynomial segments and extrapolate δ @ $u = 1$.

$$p = a_3 u^3 + a_2 u^2 + a_1 u + a_0$$
$$p^u = 3a_3 u^2 + 2a_2 u + a_1$$
$$p^{uu} = 6a_3 u + 2a_2$$

Where $p^{uu}_{u_i} = \frac{M_i L^2}{EI}$. $\quad y^{xx} = \frac{(y^u/x^u)^u}{x^u}$, $\quad y^x = \frac{y^u}{x^u}$

For $0 \le u \le u_i$:
Know $p(0), p^u(0), p^{uu}(u_i), p^{uu}(0)$

$$\begin{bmatrix} p(0) \\ p^u(0) \\ p^{uu}(0) \\ p^{uu}(u_i) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 6u_i & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

Using found coefficients.
Solve for $p^u(u_i)$, $p(u_i)$

Then, for $u_i \le u \le u_j$
Know: $p(u_i), p^u(u_i), p^{uu}(u_i), p^{uu}(u_j)$

Similar to above, solve for second set of coefficients.

Using 2nd set of coefficients, solve for $p(u_j)$ and $p^u(u_j)$.

Then, for $u_j \le u \le 1$
Know: $p(u_j), p^u(u_j), p^{uu}(u_j)$ and $p^{uu}(1)$

Similar to above, solve for 3rd set of coefficients and calculate $p(1)$.

In the end, will have the profile.
Eqns for p^u and p^{uu} will also be known.

$$p(u) = \begin{cases} \ldots & 0 \le u \le u_i \\ & u_i \le u \le u_j \\ & u_j \le u \le 1 \end{cases}$$