Sensor Placement Optimization via Parameterization

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-n Space $\vec{p} = (1-u)\vec{p}_0 + u\vec{p}_1 + \frac{1}{6}(-u^3 + 3u^2 - 2u)\vec{p}_0^{uu} + \frac{1}{6}(u^3 - u)\vec{p}_1^{uu}$

(Second derivative form, δ ← curvature)

Compact: $\vec{p} = F_1\vec{p}_0 + F_2\vec{p}_1 + F_3\vec{p}_0^{uu} + F_4\vec{p}_1^{uu}$

Say u is length along needle ($u \approx 0 - 15$cm). Reparameterize $u \in [0, 1]$
Or, is known length between two sensors.

Can we set it up as so, when we want to find $\vec{p}_1$?

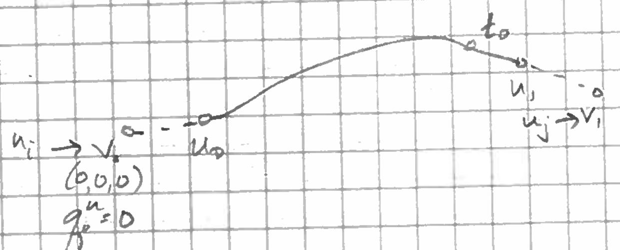
We would have to say, we know B at 2 locations (0, 1) and then extrapolate:

$$B = \begin{bmatrix} x_0 & y_0 & z_0 \\ x_1 & y_1 & z_1 \\ x_0^u & y_0^u & z_0^u \\ x_1^u & y_1^u & z_1^u \end{bmatrix}$$

Know u_0, u_1 (sensor positions)
Know $\vec{p}_0^u, \vec{p}_1^u$ from integral of strain readings
Know $\vec{p}_0$ and $\vec{p}_1$ from 2nd integral of strain.

Perhaps only $\vec{p}, \vec{p}^{uu}$ are needed for 2nd derivative form.

Vary u, vary ε, and optimize $\Delta p = |\vec{p} - p_{est}|$



Compute p_i, p_j w/ $p = UMB$
p_i^u, p_j^u w/ $p^u = UM^uB$

Perhaps v_0, u_0, u_1 could be used, to guess v_1

$g_0 = p_i$
$g_1 = p_j$
$g_0^v = (u_j - u_i)\, p_i^u$
$g_1^v = (u_j - u_i)\, p_j^u$

Could have 2 splines estimated, from $v_0 \to u_0$, $u_0 \to u_1$. Then, guess v_1, using intermediate information a little before u_1 ($t_0 \to v_1$, u_1) (for more exact slope calculation).