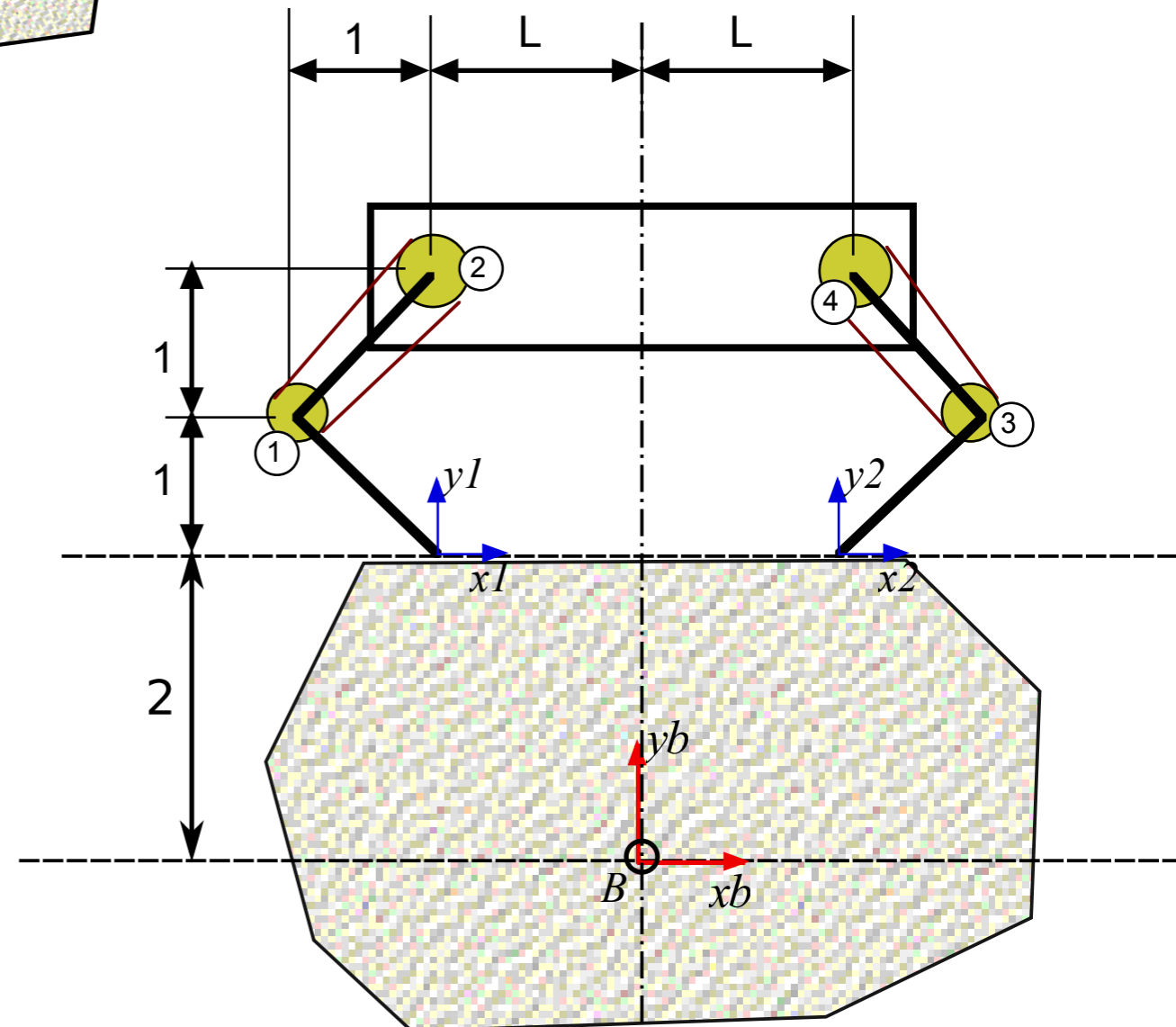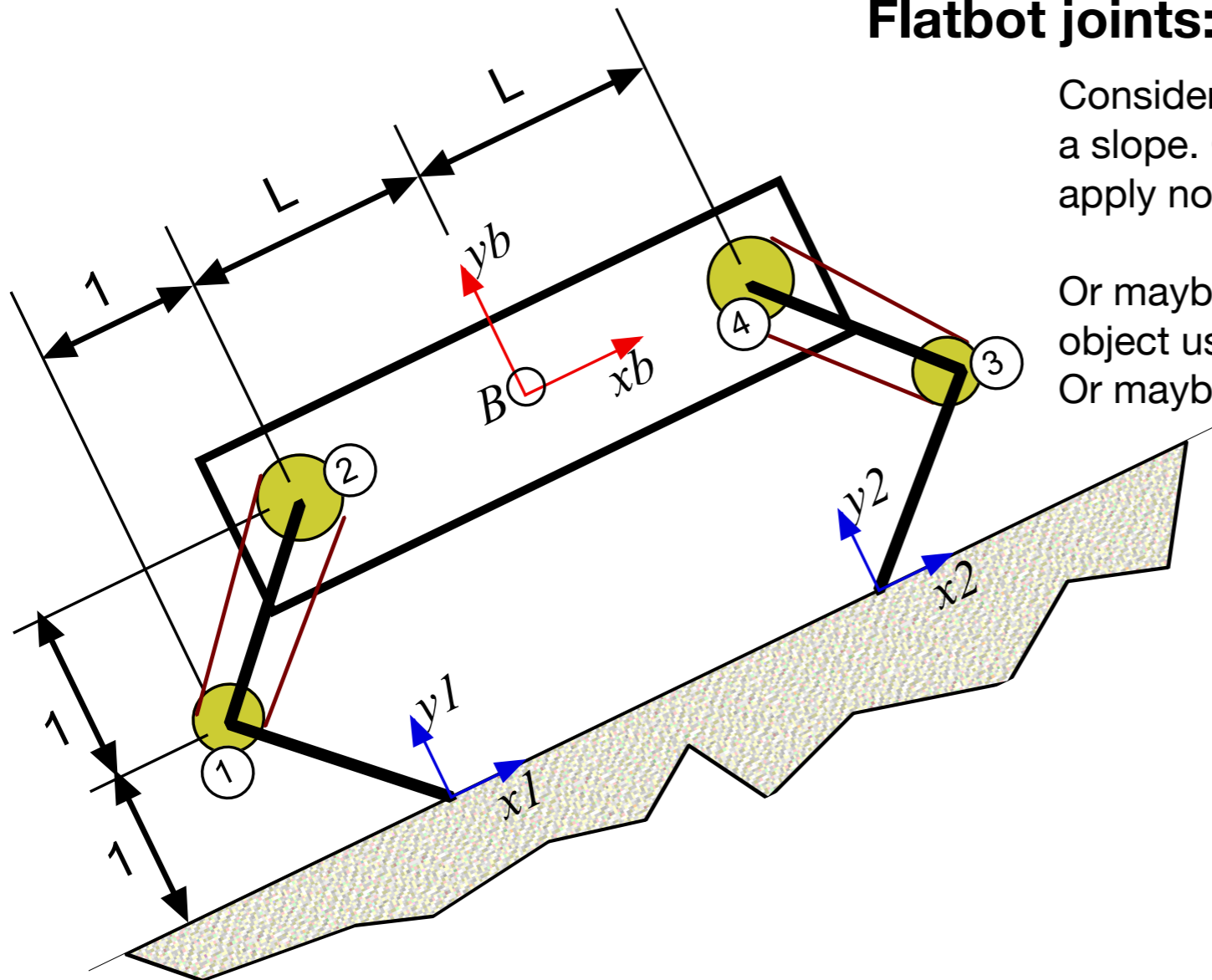## Flatbot joints:

Consider a planar model of a two-legged robot climbing a slope. Gravity acts at the center of mass, *B*. The feet apply normal and tangential contact forces to the ground.

Or maybe we have a two-fingered hand grasping a large object using spiny fingertips as shown below.
Or maybe it's a bird foot perching with toes and claws…

- How many degrees of freedom does the body have in the plane?
- How many degrees of freedom does the entire system have if both feet are in contact with the ground?
- How many independently controlled motors?
- By inspection, write the two 2x2 Jacobian matrices, J_1, J_2 that map from angular velocities at the joints (1-2, 3-4) to (x,y) motions at the feet.
- Concatentate them into a single 4x4 matrix, $\mathbf{J_J}$, that maps from $[\omega_1, \omega_2, \omega_3, \omega_4]^t$ to $[\dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2]^t$

1

Flatbot grasp:
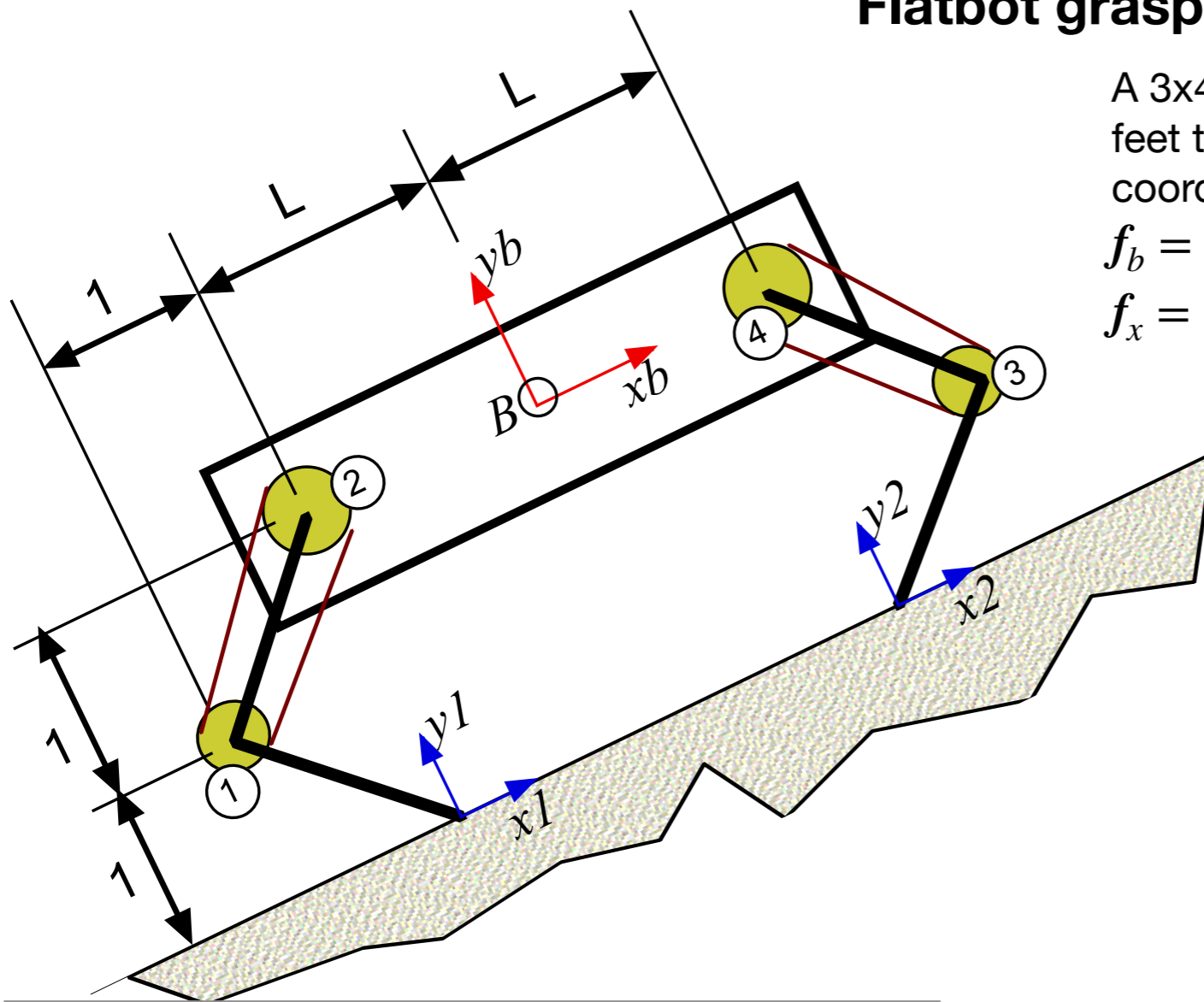
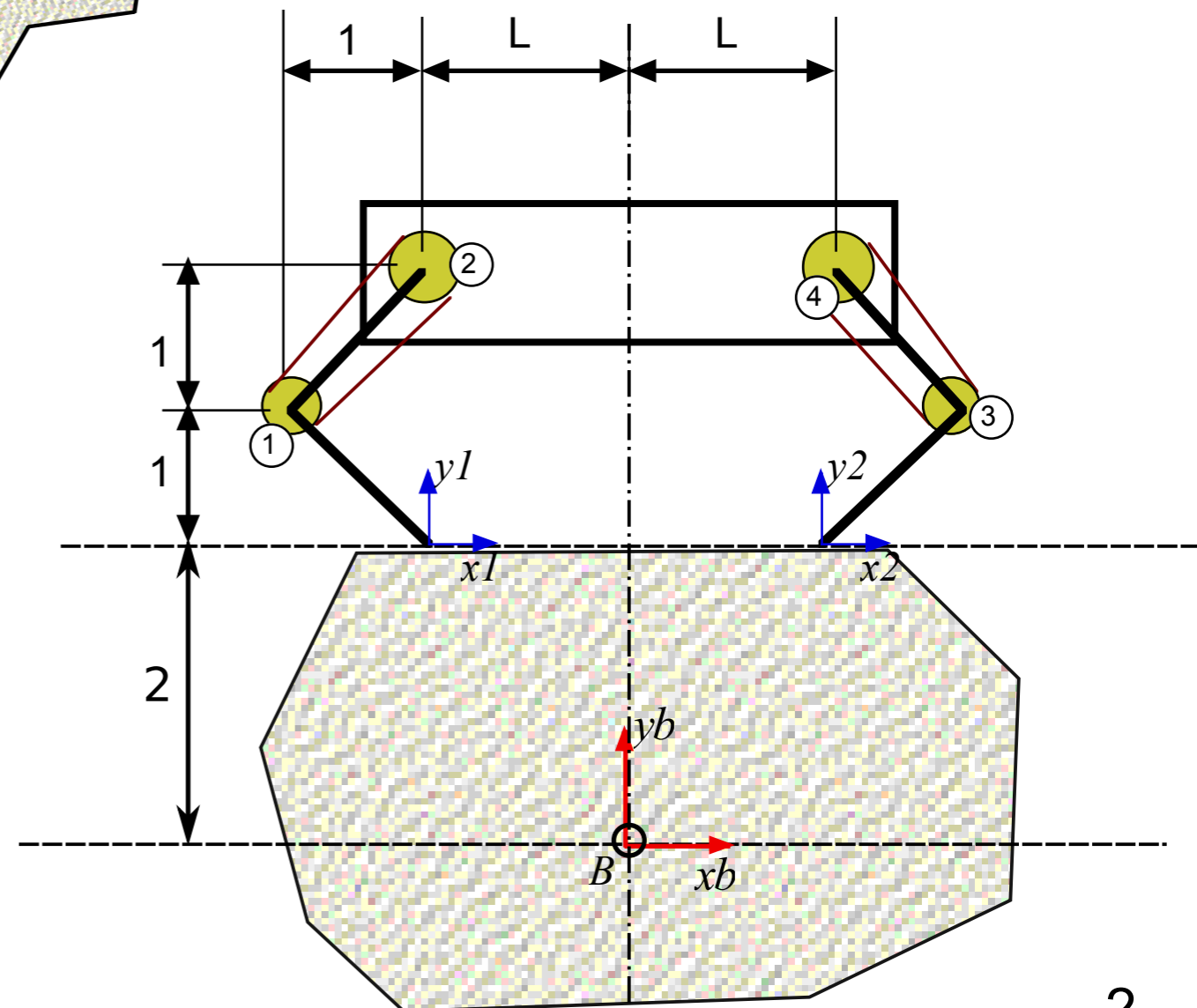A 3x4 grasp matrix, G^t, maps from forces applied by the feet to resultant forces on the body, taken at the *B* coordinate frame. That is:

$$\boldsymbol{f}_b = [G^t]\boldsymbol{f}_x, \text{ were } \boldsymbol{f}_b = [f_{xb}, f_{yb}, m_b]^t \text{ and}$$

$$\boldsymbol{f}_x = [f_{x1}, f_{y1}, f_{x2}, f_{y2}]^t \quad \text{Also, } \dot{\boldsymbol{x}}_f = [G]\dot{\boldsymbol{x}}_b$$

For the grasping example below, the 3rd row of the G^t matrix is slightly different than it is for the legged robot.

How can we invert the grasp force equation to get feet forces that will satisfy static or dynamic equilibrium requirements on the body?

- Create the $[G^t]$ matrix by inspection from feet forces, $\boldsymbol{f}_x$, to body forces, $\boldsymbol{f}_b$.
- Augment $[G^t]$ with a 4th row that maps from the feet forces to an internal force f_{int} in the null space of the external equilibrium equations. Now $\boldsymbol{f}_b = [f_{xb}, f_{yb}, m_b, f_{int}]^t$

2
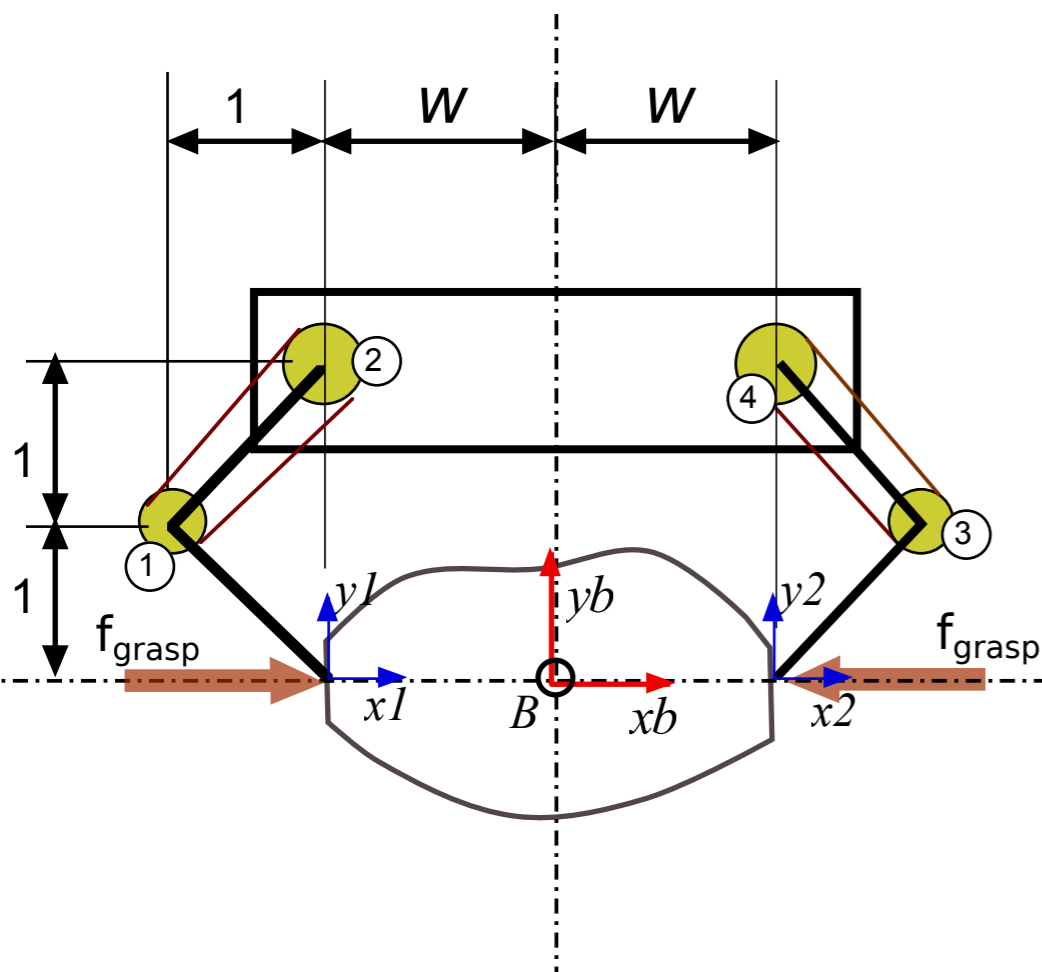
Grasp stiffness

- From a physical standpoint, what does it mean if $[K_q]$ is not diagonal?
- What would it mean if $[K_b]$ is not diagonal?
- How can we assess grasp stability? (Note that this is just one measure of a stable grasp.)

Direct Stiffness, Kb

For small displacements, $[K] = \partial f/\partial x$ and $[C] = [K]^{-1} = \partial x/\partial f$

- Like springs in series, compliances add along a serial chain.
- Like springs in parallel, stiffnesses add for multiple chains connected to a common object.

We can map from joint stiffness (due to servo gains and/or passive torsional springs) to an equivalent fingertips stiffness:

1. $df_x = K_x dx$ or $dx = C_x df_x$. Similarly $dq = C_q d\tau$ in joint space

   where $dx = [dx_1, dy_1, dx_2, dy_2]^t$ and $\tau = [\tau_1, \tau_2, \tau_3, \tau_4]^t$, etc. Note that we have not put an object in yet, so there is no restriction on dx.

2. Using conservation of virtual work: $f_x^t \cdot dx = \tau^t \cdot dq$.

3. Combining 1,2: $f_x^t \cdot (C_x df_x) = \tau^t \cdot (C_q d\tau)$

4. Recall $dx = J_J dq$ and $\tau = J_J^t f_x$

5. Combining 3,4: $f_x^t C_x df_x = f_x^t J_J C_q J_J^t df_x$ so $C_x = J_J C_q J_J^t$
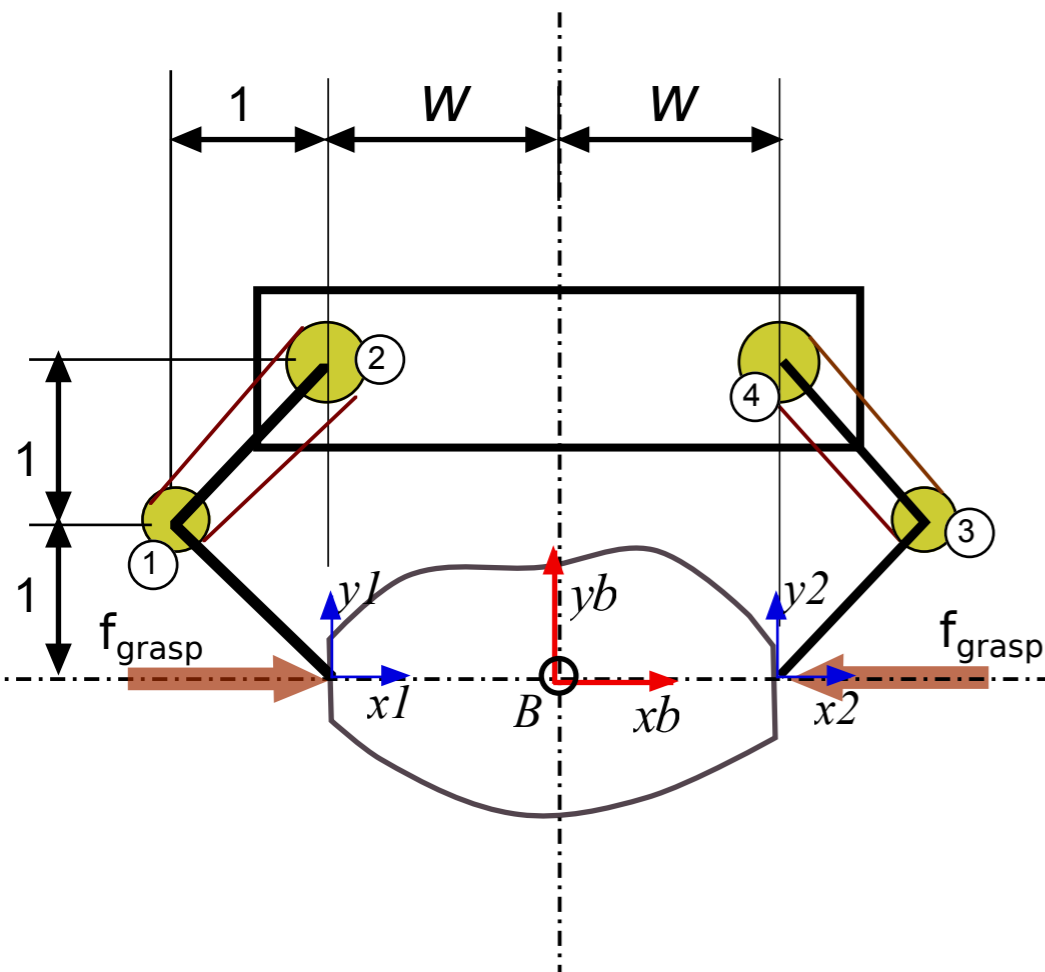
6. Finally, $K_x = C_x^{-1}$

We can extend this reasoning to the grasped body, again using virtual work: $dx_b^t \cdot (K_b dx_b) = dx_f^t \cdot (K_x dx_f)$ where $dx_b = [dx, dy, d\theta]^t$

Recall $G dx_b = dx_f$, so $(dx_b^t G^t) \cdot K_x \cdot G dx_b$

or $K_b = G^t K_x G$.

We do not include an internal grasp term, so K_x is 4x4 but K_b is 3x3.

3

Grasp stiffness cont'd.
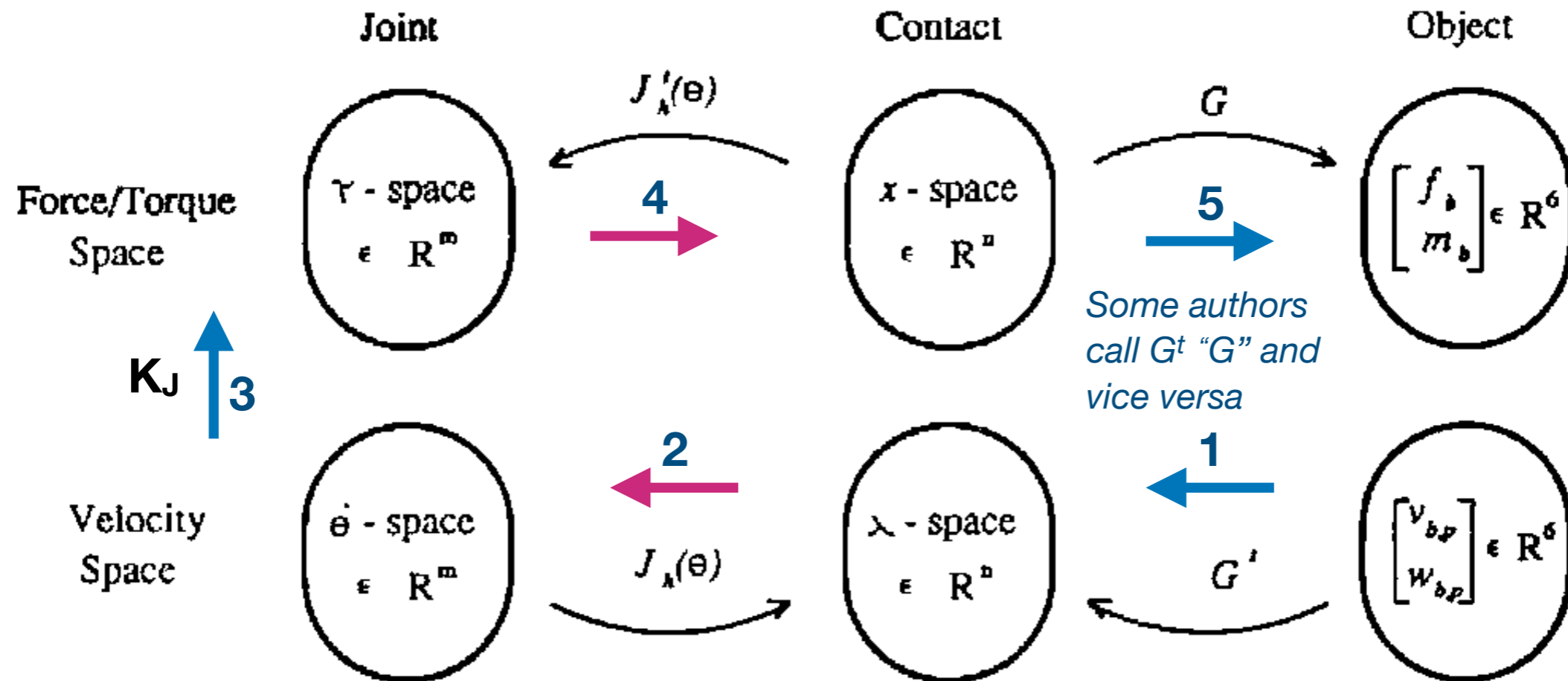

Geometric effect, K_{geom}

A second, geometric effect contributes to the effective stiffness and stability of a grasp.

- Let f_{grasp} be the internal grasp force applied by the fingers.
- At equilibrium the grasp forces produce no net external force or moment on the body (i.e., they are in the homogeneous solution to the force balance equations).
- But if the body rotates slightly, $d\theta$, they become misaligned and produce a net torque: $dm = 2f_{grasp}wd\theta$.
- This torque tends to increase $d\theta$ so it is *destabilizing*.
- We combine such effects to create a geometric stiffness matrix, $[K_{geom}]$ which, unlike $[K_b]$, may have negative terms and may be destabilizing.
- So finally, $[K_{total}] = [K_b] + [K_{geom}]$

The details of $[K_{geom}]$ will depend on the contact type (e.g. whether rolling or point contacts). In the spreadsheet example we consider only the most important term for a grasp with point contacts.

For physical intuition, it may help to imagine that the unit of length is 1 decimeter (0.1m) so if $kq_{ii} = 1$, that's 0.1 Nm/radian and so forth.

- If $w = 1$ and all $kq_{ii} = 1$ in joint space, how big can f_{grasp} be before the grasp is unstable?
- Does increasing w make things better or worse? Why?
- Can adding some off-diagonal terms, say $kq_{ij} = \pm 0.5$, make things better?

The kinematic loop:

| Joint | Contact | Object |

Force/Torque Space — Υ - space $\epsilon\ R^m$ — $J'_\lambda(\theta)$ — **4** — x - space $\epsilon\ R^n$ — **5** — G — $\begin{bmatrix} f_b \\ m_b \end{bmatrix} \epsilon\ R^6$

K_J **3**

Some authors call G^t "G" and vice versa

Velocity Space — $\dot\theta$ - space $\epsilon\ R^m$ — **2** — $J_\lambda(\theta)$ — λ - space $\epsilon\ R^n$ — **1** — G' — $\begin{bmatrix} v_{b,p} \\ w_{b,p} \end{bmatrix} \epsilon\ R^6$

Li, Hsu, Sastry, "Grasping and Coordinated Manipulation by a Multifingered Robot Hand," IJRR, 1989; 8; 33 (similar diagrams in Cutkosky1985, Okamura et al. 2000).

Going from joint stiffness (eg servo stiffness) to equivalent object stiffness requires traversing the whole path from lower right to upper left; some parts are forward, some are inverse.

small object motion => small joint motions => change in joint forces => change in object forces

Homework (due tuesday Jan 14)

- Review these slides and play with the spreadsheet
- Answer the corresponding Force and Stiffness questions on Canvas: https://canvas.stanford.edu/courses/114293
- Read the articles for Week 2, which cover some of the same topics in a more general way with 6 DoF wrenches and twists.